Section 7.6: Transhipment

SPELLING NOTE: "Transhipment" may be spelled with either 1 s or 2. I think it looks better with one, but our author uses 2. Your choice...

In a transhipment problem, we introduce points that are like a way station or distribution warehouse- Goods are shipped there to be stored and shipped out later.

Now we have three kinds of nodes:

- Supply Nodes: Only have outgoing arrows.

- Demand Nodes: Only have incoming arrows.

- Transhipment Nodes: Have both incoming and outgoing arrows.

Before proceeding, let's assume a balanced transportation problem, so add a dummy demand point with a supply of 0 and demand equal to the excess demand, before we start. Then we define s =total available supply. Now, the trick in this probleml is to determine what the supply/demand (or input/output) ought to be for the transhipment points.

- Supply: These nodes could possibly handle the entire net's worth of supply. Therefore, the supply of the transshipment node should be:

$$\text{Original Supply Value} + s$$

- Demand: Similarly, these nodes could theoretically each take in the entire supply, so we set the demand at the transshipment node:

$$\text{Original Demand Value} + s$$

Example:

Suppose we have three factories, two warehouses and three retail locations. Factories may move material to the two warehouses, but only the third factory may deliver goods directly to the third retail location. Otherwise, the retail locations receive all goods from the two warehouses.

From this, we get the transportation table. The table was initially set up with the idea that every input was connected to every output- We will use our "big-M" to deal with connections that should not be used. For example, the connection from Warehouse 1 to Retail 6, 7, 8 would each have a cost of M, a very very large number.

Since transhipment points have a slot in both the supply and demand categories, they may "ship" to themselves. This would correspond to keeping supplies in the warehouse, and unless specified otherwise, would be assumed to have zero cost.

The table is given below, where we indicate the index of each node on the side/top. From this, we know how to write the associated linear program- We might make the exception that cells with an M correspond to variables that are not used, so the LP in this case would not actually have 25 variables, but rather only 15 variables (we have 10 M's below).

Further, using the network diagram actually allows us to just write one constraint per node. After this diagram, we'll discuss the LP in more detail.
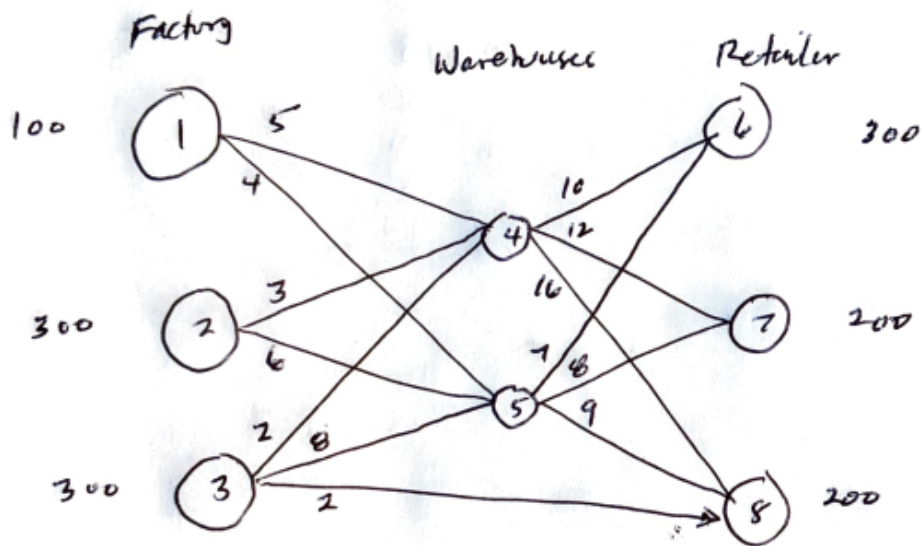
	4	5	6	7	8	Supply
1	5	4	M	M	M	100
2	3	6	M	M	M	300
3	2	8	M	M	2	300
4	0	M	10	12	16	700
5	M	0	7	8	9	700
Demand	700	700	300	200	200	

The solution to this problem is given on the next page. It is very straightforward to code this up in a spreadsheet (Excel or LibreOffice). The sample shown is linked from our class webpage.

Transhipment Example from Class (3 Factories, 2 Warehouses, 3 Retail Outlets)

		W4	W5	R6	R7	R8		
Value of s		700						
Value of M		5000						
		W4	W5	R6	R7	R8		
Cost Matrix	F1	5	4	5000	5000	5000		
	F2	3	6	5000	5000	5000		
	F3	2	8	5000	5000	2		
	W4	0	5000	10	12	16		
	W5	5000	0	7	8	6		
Cells for Xij	F1	0	100	0	0	0	100	100
	F2	200	100	0	0	0	300	300
	F3	100	0	0	0	200	300	300
	W4	400	0	300	0	0	700	700
	W5	0	500	0	200	0	700	700
		700	700	300	200	200		
		700	700	300	200	200		
Objective Function:		6800						

You might note that in our table, we're allowing the warehouses to ship to themselves. That is actually kind of critical here, because of the way we're computing input-output for the transhipment points. We're taking the biggest possible value of input and output (in how we're computing s). There is probably a more effective way of computing the LP in this problem- A method where we go from cell to cell in the network diagram, and for each cell, we just need to make sure that **input** = **output**. If you look at our optimal solution given by the spreadsheet, you will notice that this is true of this solution. For example, we compute the equations for some of the nodes below:

- Input Node 1: $100 = x_{14} + x_{15}$

  This equation would be the same in the standard LP formulation as row sum 1.

- Transhipment Node 4: $x_{14} + x_{24} + x_{34} = x_{46} + x_{47} + x_{48}$

  This is interesting- In the standard LP formulation, we would take $x_{14} + x_{24} + x_{34} + x_{44} = 700$, and similarly, $x_{44} + x_{46} + x_{47} + x_{48} = 700$, but now we're making these two constraints into a single constraint by removing x_{44}, and we also no longer need to compute some upper bound value for s.

- Output Node 8: $x_{38} + x_{58} + x_{48} = 200$

  This equation would be the same in the standard LP formulation as col sum 5.

Example for You to Try

Given the network diagram below, (i) first write the transhipment problem as a transportation problem, (ii) solve it using a spreadsheet, and (iii) write the LP using the suggested method above, where we have one equation per cell in the network, and we make input equal output at each node (so you'll have 7 constraints).
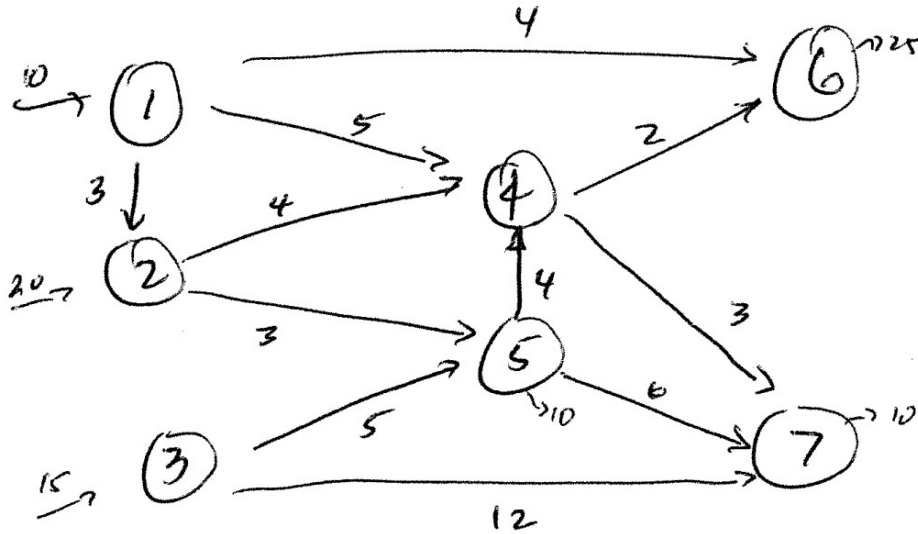
(Solution is on the next page)

SOLUTION: Since we aren't writing the x_{ij} in, we'll just write the costs down. Next to the cost matrix is the solution using a spreadsheet. The value of s below is the common supply/demand value, which in this case is $s = 45$. We leave it as s in the table below so you can see what computations we are making- In particular, Node 2 is being treated as a transhipment point rather than as a "pure" supply node.

	2	4	5	6	7	Supply
1	3	5	M	6	M	10
3	M	M	5	M	12	15
2	0	4	3	M	M	$20 + s$
4	M	0	M	2	3	s
5	M	4	0	M	6	s
Demand	s	s	$10 + s$	25	10	

Solution from the spreadsheet:

	2	4	5	6	7	
1				10		10
3			15			15
2	45	20				$20 + s$
4		25		15	5	s
5			40		5	s
	s	s	$10 + s$	25	10	

For the LP, we can disregard variables in the cells with M, so we have 14 variables. We want to minimize the following, given the constraints following:

$$3x_{12} + 5x_{14} + 6x_{16} + 5x_{35} + 12x_{37} + 4x_{24} + 3x_{25} + 2x_{46} + 3x_{47} + 4x_{54} + 6x_{57}$$

The constraints below are for cells numbered 1-7 respectively.

$10 = x_{12} + x_{14} + x_{16}$

$20 + x_{12} = x_{24} + x_{25}$

$15 = x_{35} + x_{37}$

$x_{14} + x_{24} + x_{54} = x_{46} + x_{47}$

$x_{25} + x_{35} = x_{54} + x_{57} + 10$

$x_{16} + x_{46} = 25$

$x_{47} + x_{57} + x_{37} = 10$