## Homework, Section 7.3

1. Given transportation tableau below, write out the original linear program and its dual (be sure to use $u, v$ notation for the dual).

|  |  | 2 |  | 3 |  | 5 |  | 6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 |  | 1 |  | 3 |  | 5 |  |
|  |  | 3 |  | 8 |  | 4 |  | 6 |  |
| Demand | 12 |  | 8 |  | 4 |  | 6 |  | 30 |

2. Given the BFS below, compute the solution to the dual and determine if the BFS gives the optimal solution. If not, say which cell should come into the basis.

|  | $v_{1}=$ |  | $v_{2}=$ |  | $v_{3}=$ |  | $v_{4}=$ |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 |  | 3 |  | 5 |  | 6 |  |
| $u_{1}=$ | 5 |  |  |  |  |  |  |  | 5 |
|  |  | 2 |  | 1 |  | 3 |  | 5 | 10 |
| $u_{2}=$ | 7 |  | 3 |  |  |  |  |  |  |
|  |  | 3 |  | 8 |  | 4 |  | 6 |  |
| $u_{3}=$ |  |  | 5 |  | 4 |  | 6 |  | 15 |
| Demand | 12 |  | 8 |  | 4 |  | 6 |  | 30 |

3. Continuing from the previous answer, update the tableau and check if it is optimal:

|  | $v_{1}=$ |  | $v_{2}=$ |  | $v_{3}=$ |  | $v_{4}=$ |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $u_{1}=$ |  | 2 |  | 3 |  | 5 |  | 6 | 5 |
|  |  |  |  |  |  |  |  |  |  |
|  |  | 2 |  | 1 |  | 3 |  | 5 | 10 |
| $u_{2}=$ |  |  |  |  |  |  |  |  |  |
|  |  | 3 |  | 8 |  | 4 |  | 6 |  |
| $u_{3}=$ |  |  |  |  |  |  |  |  | 15 |
| Demand | 12 |  | 8 |  | 4 |  | 6 |  | 30 |

4. In the previous problem, at least one cell should have (0), which means that entry could be used in the BFS, which would give an alternate BFS- Find an alternate BFS to the one you found above.
5. Here is a transportation tableau. Use any method to get a BFS, then use MODI to find the optimal solution.

|  | $v_{1}=$ |  | $v_{2}=$ |  | $v_{3}=$ |  | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $u_{1}=$ |  | 2 |  | 2 |  | 3 | 10 |
|  |  |  |  |  |  |  |  |
|  |  | 4 |  | 1 |  | 2 |  |
| $u_{2}=$ |  |  |  |  |  |  | 15 |
|  |  | 1 |  | 2 |  | 1 |  |
| $u_{3}=$ |  |  |  |  |  |  | 40 |
| Demand | 20 |  | 15 |  | 30 |  | 65 |

6. If we have 4 warehouses and 5 customers, (i) How many decision variables are in the original linear program for the transporation problem? (ii) How many variables need to be determined to make up a basic feasible solution? Are there any other restrictions on those variables (to make up a BFS)?
