Sensitivity Analysis

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- Changes to a cost for a NBV.

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- Changes to a cost for a NBV.
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Sensitivity Analysis

We consider:

- Changes to a cost for a NBV.
- Changes to a cost for a BV.
- Change to a supply/demand pair.

Change to a cost for a NBV

- If c_{ij} changes to $c_{ij} + \Delta$, then u_i, v_j

Change to a cost for a NBV

- If c_{ij} changes to $c_{ij} + \Delta$, then u_i, v_j will not change (we used BVs for those).

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- Only change is to "Row 0" in cell (i, j):

Change to a cost for a NBV

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- Only change is to "Row 0" in cell (i, j):

$$(c_{ij} + \Delta) - (u_i + v_j) =$$

Change to a cost for a NBV

- If c_{ij} changes to $c_{ij} + \Delta$, then u_i, v_j will not change (we used BVs for those).
- Only change is to "Row 0" in cell (i, j):

$$(c_{ij} + \Delta) - (u_i + v_j) = (c_{ij} - (u_i + v_j)) + \Delta$$

Example 1

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	30	125

Example 1

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	$8 + \Delta$	6	10	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	30	125

Example 1

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	$8 + \Delta$ $(2 + \Delta)$	6 10	10 25	9	35
$u_2 = 3$	9 45	12	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	20	30	30	125

Example 1

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	$8 + \Delta$ $(2 + \Delta)$	6 10	10 25	9	35
$u_2 = 3$	9 45	12	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	20	30	30	125

Current basis is optimal as long as $\Delta > -2$.

Example 2

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	$16 + \Delta$	5	
		10		30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	$16 + \Delta$	5	
		10	$(3 + \Delta)$	30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	$16 + \Delta$	5	
		10	$(3 + \Delta)$	30	40
Demand	45	20	30	30	125

Current basis is optimal for $\Delta > -3$.

Changing a basic variable could change many of the u_i, v_j. We will have to re-compute them, and all of the other "Row 0" changes...

Example 3: Change cost for a BV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	$10 + \Delta$	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	$10 + \Delta$	9	
		10	25		35
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	$10 + \Delta$	9	
		10	25		35
$u_2 = 3 - \Delta$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	$10 + \Delta$	9	
		10	25		35
$u_2 = 3 - \Delta$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
	8	6	$10 + \Delta$	9	
$u_1 = 0$		10	25		35
	9	12	13	7	
$u_2 = 3 - \Delta$	45		5		50
	14	9	16	5	
$u_3 = 3$		10		30	40
Demand	45	20	30	30	125

Now we compute the other "Row 0" values...

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12 $(3 + \Delta)$	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12 $(3 + \Delta)$	13 5	7 $(2 + \Delta)$	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12 $(3 + \Delta)$	13 5	7 $(2 + \Delta)$	50
$u_3 = 3$	14 $(5 - \Delta)$	9 10	16	5 30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12 $(3 + \Delta)$	13 5	7 $(2 + \Delta)$	50
$u_3 = 3$	14 $(5 - \Delta)$	9 10	16 $(3 - \Delta)$	5 30	40
Demand	45	20	30	30	125

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12 $(3 + \Delta)$	13 5	7 $(2 + \Delta)$	50
$u_3 = 3$	14 $(5 - \Delta)$	9 10	16 $(3 - \Delta)$	5 30	40
Demand	45	20	30	30	125

Five inequalities must remain satisfied in order for the current basis to remain optimal:

$$2 - \Delta > 0, \quad 3 + \Delta > 0, \quad 2 + \Delta > 0, \quad 5 - \Delta > 0, \quad 3 - \Delta > 0$$

Example 2

	$v_1 = 6 + \Delta$	$v_2 = 6$	$v_3 = 10 + \Delta$	$v_4 = 2$	Supply
$u_1 = 0$	8 $(2 - \Delta)$	6 10	$10 + \Delta$ 25	9 (7)	35
$u_2 = 3 - \Delta$	9 45	12 $(3 + \Delta)$	13 5	7 $(2 + \Delta)$	50
$u_3 = 3$	14 $(5 - \Delta)$	9 10	16 $(3 - \Delta)$	5 30	40
Demand	45	20	30	30	125

Five inequalities must remain satisfied in order for the current basis to remain optimal:

$$2 - \Delta > 0, \quad 3 + \Delta > 0, \quad 2 + \Delta > 0, \quad 5 - \Delta > 0, \quad 3 - \Delta > 0$$

so that

$$-2 < \Delta < 2$$

Change to s_i, d_j

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	$35 + \Delta$
		10	25		
$u_2 = 3$	9	12	13	7	50
	45		5		
$u_3 = 3$	14	9	16	5	40
		10		30	
Demand	45	20	30	30	125

Change to s_i, d_j

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	$35 + \Delta$
		10	25		
$u_2 = 3$	9	12	13	7	50
	45		5		
$u_3 = 3$	14	9	16	5	40
		10		30	
Demand	45	20	30	30	125

To stay balanced, a demand must also increase by Δ.
Suppose we choose the second demand (so that cell $(1, 2)$ is basic)

Change to s_i, d_j, Cell (i, j) is Basic

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6 10	10 25	9	$35 + \Delta$
$u_2 = 3$	9 45	12	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	$20 + \Delta$	30	30	125

Change to s_i, d_j, Cell (i, j) is Basic

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6 $10 + \Delta$	10 25	9	$35 + \Delta$
$u_2 = 3$	9 45	12	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	$20 + \Delta$	30	30	125

Change to s_i, d_j, Cell (i, j) is Basic

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6 $10 + \Delta$	10 25	9	$35 + \Delta$
$u_2 = 3$	9 45	12	13 5	7	50
$u_3 = 3$	14	9 10	16	5 30	40
Demand	45	$20 + \Delta$	30	30	125

The change was completely localized.
The change in z will be:

Change to s_i, d_j, Cell (i, j) is Basic

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		$10 + \Delta$	25		$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	$20 + \Delta$	30	30	125

The change was completely localized.

The change in z will be: an increase of 6Δ

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25		$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	$30 + \Delta$	125

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25	Δ	$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	$30 + \Delta$	125

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25	Δ	$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	$30 + \Delta$	125

This creates a loop!

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25	Δ	$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	$30 + \Delta$	125

This creates a loop! (Linear dependency)

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		10	25	Δ	$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		10		30	40
Demand	45	20	30	$30 + \Delta$	125

10	Δ
10	30

$$\begin{array}{c|c} 10 & \Delta \\ \hline 10 & 30 \end{array} \quad \rightarrow \quad \begin{array}{c|c} 10+\Delta & \Delta-\Delta \\ \hline 10-\Delta & 30+\Delta \end{array}$$

$$\begin{array}{c|c} 10 & \Delta \\ \hline 10 & 30 \end{array} \rightarrow \begin{array}{c|c} 10+\Delta & \Delta-\Delta \\ \hline 10-\Delta & 30+\Delta \end{array} \rightarrow \begin{array}{c|c} 10+\Delta & \\ \hline 10-\Delta & 30+\Delta \end{array}$$

$$\begin{array}{c|c} 10 & \Delta \\ \hline 10 & 30 \end{array} \quad \rightarrow \quad \begin{array}{c|c} 10+\Delta & \Delta-\Delta \\ \hline 10-\Delta & 30+\Delta \end{array} \quad \rightarrow \quad \begin{array}{c|c} 10+\Delta & \\ \hline 10-\Delta & 30+\Delta \end{array}$$

Corresponding to the new tableau following:

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	$35 + \Delta$
		$10 + \Delta$	25		
$u_2 = 3$	9	12	13	7	50
	45		5		
$u_3 = 3$	14	9	16	5	40
		$10 - \Delta$		$30 + \Delta$	
Demand	45	20	30	$30 + \Delta$	125

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	$35 + \Delta$
		$10 + \Delta$	25		
$u_2 = 3$	9	12	13	7	50
	45		5		
$u_3 = 3$	14	9	16	5	40
		$10 - \Delta$		$30 + \Delta$	
Demand	45	20	30	$30 + \Delta$	125

We want all these cells to remain positive...

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		$10 + \Delta$	25		$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		$10 - \Delta$		$30 + \Delta$	40
Demand	45	20	30	$30 + \Delta$	125

We want all these cells to remain positive...

Change in z:

Change to s_i, d_j, Cell (i, j) is a NBV

	$v_1 = 6$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$	Supply
$u_1 = 0$	8	6	10	9	
		$10 + \Delta$	25		$35 + \Delta$
$u_2 = 3$	9	12	13	7	
	45		5		50
$u_3 = 3$	14	9	16	5	
		$10 - \Delta$		$30 + \Delta$	40
Demand	45	20	30	$30 + \Delta$	125

We want all these cells to remain positive...

Change in z: $+6\Delta - 9\Delta + 5\Delta = +2\Delta$