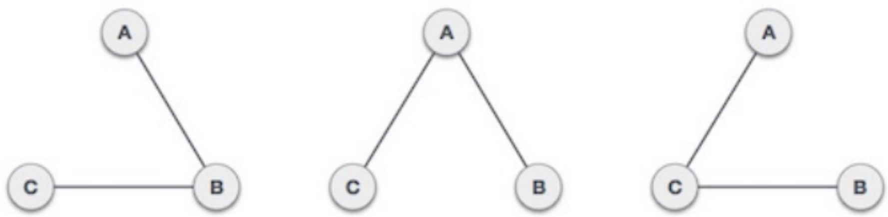
8.6: Minimum Spanning Tree

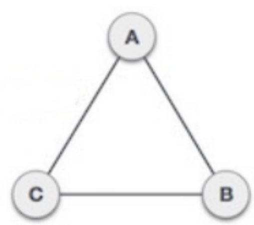
For a network with n nodes, a **spanning tree** is a group of $n - 1$ arcs that connects all nodes of the network and contains no loops.

Example

Consider the graph G with nodes A, B, C. Then any of the following would represent a spanning tree:



And we should note that the following would **NOT** be a spanning tree, because it has 3 arcs and contains a loop.

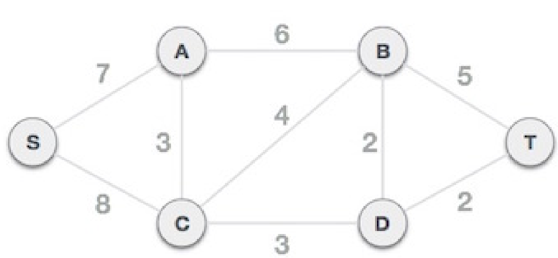


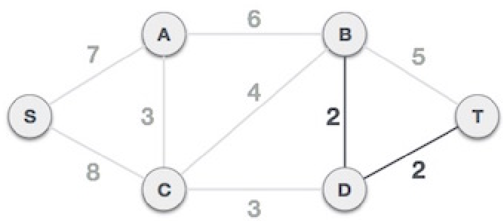
If the arcs have costs (or values that represent a distance between nodes as another common kind of problem), then a typical problem is to find a spanning tree that minimizes the sum of the costs (or distances).

Two common algorithms for finding the minimum spanning tree are *Prim's Algorithm* and *Kruskal's Algorithm.* Although our book does not mention it by name, it uses Kruskal's Algorithm. The basic idea is straightforward:

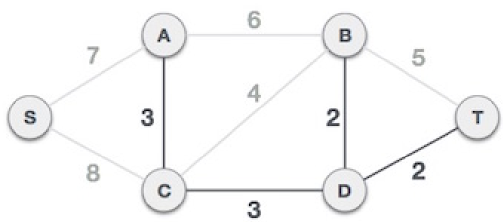
> Given the graph G, add the edges with the minimum cost that won't otherwise violate the properties of a spanning tree (that is, no loops).

Let's see how this works in practice. Here is our initial network.

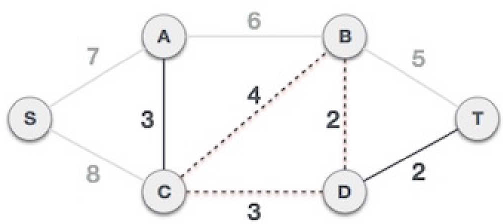




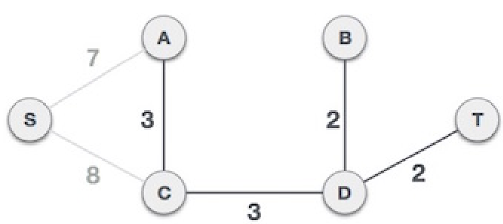
Step 1: Find the arc with the minimum value, and include it in the spanning tree. In this case, we have two arcs with the lowest, and including them both does not create a loop.



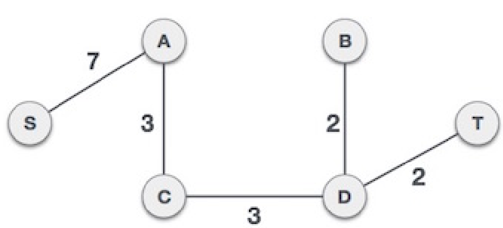
Step 2: The next lowest arc has value 3, and again we have two such arcs. Including them both does not create a loop, so we'll keep both.



Step 3: The next lowest arc has value 4, but including that edge creates a loop, so we discard that one and continue.



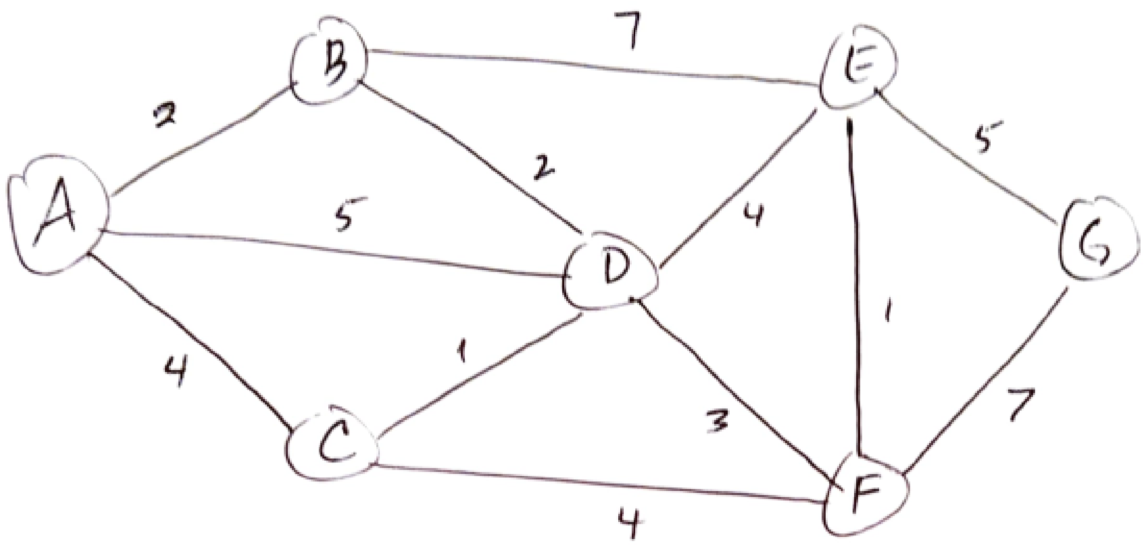
Step 4: The next lowest arc has value 5- including it creates a loop, as does the edge with value 6. We'll remove those from consideration.

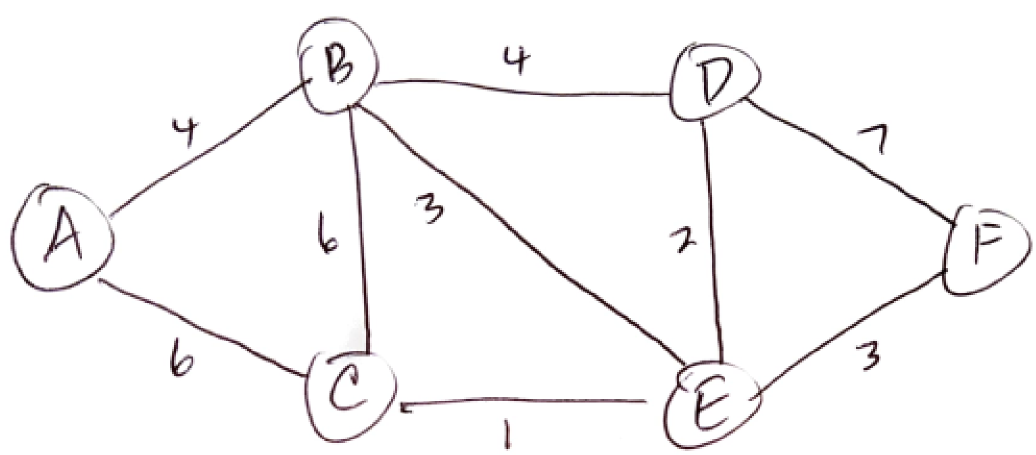


Step 5: The last edge has value 7. Including that gives us a tree with 5 edges (we have 6 nodes), and it is a minimal spanning tree.

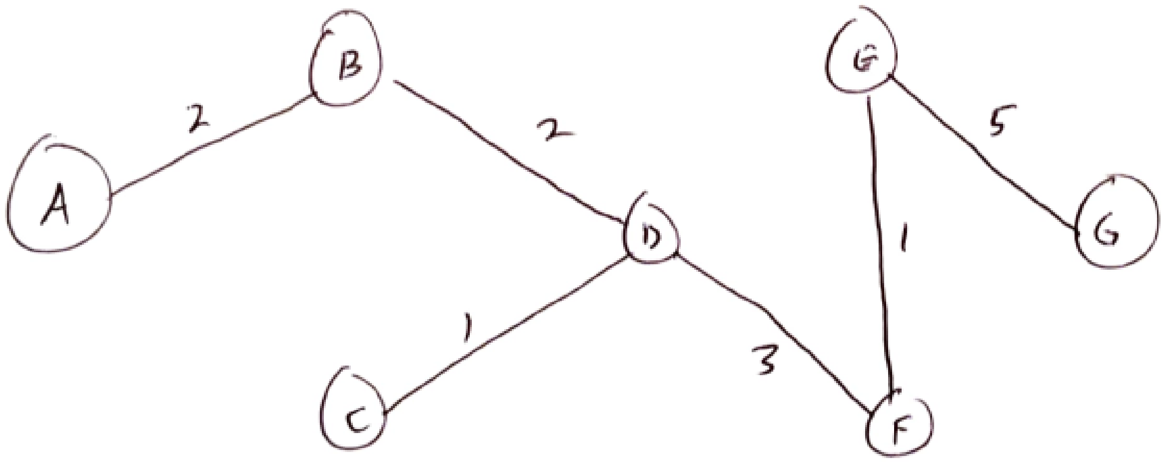
Two Worked Examples

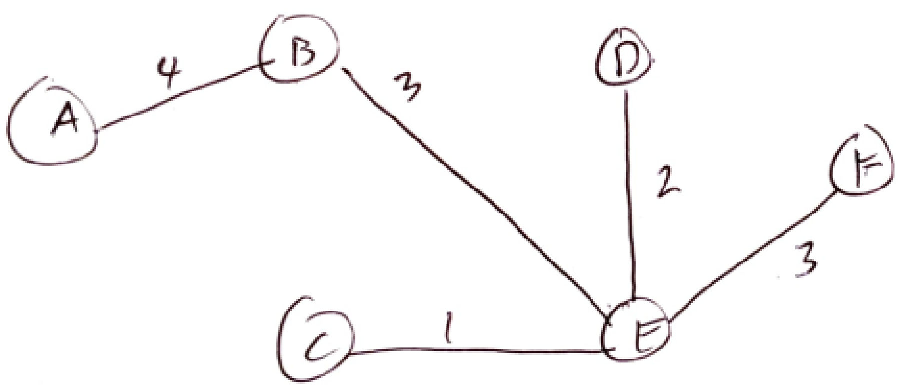
Try working out the minimum spanning tree for these two examples. The solutions are on the next page (try to do it without peeking!)





Solutions to the previous page

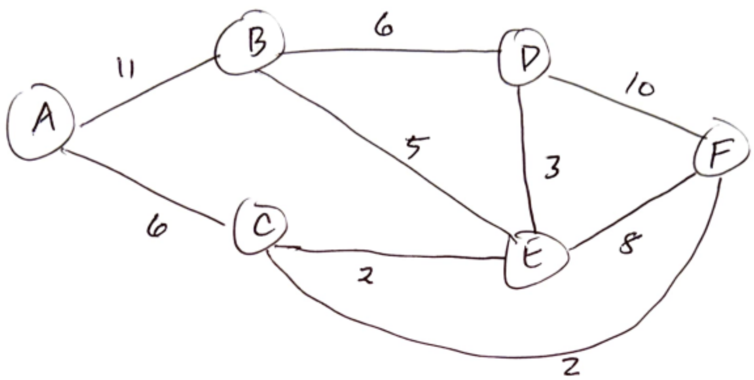




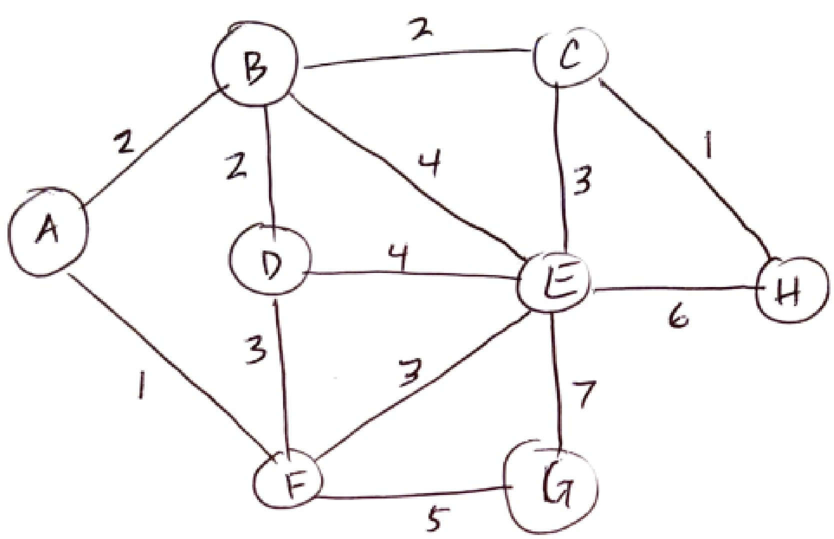
8.6 Exercises

For the following graphs, find the minimum spanning tree.

1. Graph 1:



2. Graph 2:



3. Graph 3: