In-class problems

1. Let $W = \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}\right\}$, $\mathbf{y} = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$.

   - Project $\mathbf{y}$ to each of the basis vectors of W.

   - What is the orthogonal projection of $\mathbf{y}$ onto W, $\hat{\mathbf{y}}$

   - Compute $\mathbf{z} = \mathbf{y} - \hat{\mathbf{y}}$. What should this be orthogonal to?

2. Let S be a subspace of $\mathbb{R}^n$, and let the columns of X form a basis for S, so that X is $n \times k$.

   Show that the matrix $P = X(X^T X)^{-1} X^T$ is the projection matrix to S by doing the following:

   - Show that $P^2 = P$
   - Show that, for any $\mathbf{y} \in \mathbb{R}^n$, $\mathbf{y} - P\mathbf{y} \perp S$. You can do this by showing that the dot product between every basis vector in S and the vector $\mathbf{y} - P\mathbf{y}$ is 0, or that

   $$X^T(\mathbf{y} - P\mathbf{y}) = \vec{0}$$