Worksheet: Feedforward Neural Nets

This worksheet will walk you through the process of computing using a feedforward neural net. Be sure to make each computation and show your answers.

1. Suppose we have a $2-2-1$ network.

   Input vector x with weights and bias terms are given below.

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad W^{(1)} = \begin{bmatrix} 1 & -1 \\ 1/2 & 1 \end{bmatrix}, \quad \mathbf{b}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} 1 & -1 \end{bmatrix}, \quad b^{(2)} = 0$$

   (a) Make a forward pass through the network, computing the prestate, state and derivative values along the way. In this case, the transfer function is ReLU, and it is only applied at the hidden layer (so the output transfer function is the identity.

2. Backpropagation Practice: Use the network from Exercise 1 with the following values:

   - Target output $t = 1$
   - Error (or "Loss"): $E = \frac{1}{2}(t-y)^2$
   - Learning rate $\alpha = 1/10$

   (a) Compute the Δ vectors for the output layer and hidden layer.

   (b) Compute $\partial E/\partial W_{ij}^{l}$ for each weight in $W^{(1)}, W^{(2)}$.

   (c) Update the weights and bias terms:

3. Example of Vanishing Gradient

   (a) Construct a $2-3-3-1$ network (this one has two hidden layers), and use the "logsig" sigmoidal: $\sigma(r) = 1/(1+\mathrm{e}^{-z})$.

   (b) Initialize all weights to 0.01 and compute the forward pass for $x = (1,1)$.
   NOTE: You can use Octave-online or Python like a calculator for this to speed up computations, but you should make the calculations like you did in Exercise 1.

   (c) Perform backpropagation and track how small gradients become in earlier layers.

4. Given a ReLU neuron with weights $\mathbf{w} = [-4,-5]$ and bias $b = -2$, compute the prestate P and the activated output $S = \mathrm{ReLU}(P)$ for the following inputs:

   (a) $\mathbf{x}_1 = (1,1)$ (b) $\mathbf{x}_2 = (0,2)$ (c) $\mathbf{x}_3 = (3,-1)$

   Determine if the neuron is suffering from the dying ReLU problem for these inputs.