Matlab and Linear Algebra, Week 4

Lab Examples

Our tasks during the lab:

1. Perform an example of linear regression. Here is a quick script to get some data (so that the line is $3x + 2$ with added noise).

   ```
   x=randn([20,1]); x=sort(x);
   y=3*x+2+randn(size(x));
   ```

   Once you get the slope and intercept, plot the data and the line. If the slope is m and the intercept is b, then this would be typed as:

   ```
   x2=linspace(min(x),max(x));
   yout=m*x2+b;
   plot(x,y,'*',x2,yout,'-');
   ```

2. Show by means of an example that the mean of the projection is the projection of the mean. Here's a short script for some data that we'll store in matrix X. Also, let the vector $\vec{u} = [1/\sqrt{2}, 1/\sqrt{2}]^T$.

   ```
   X=3*rand([120,2])+[5,7];
   ```

   Then:

   - Compute the mean of X (as a vector in $\mathbb{R}^2$).
   - Project the mean onto $\vec{u}$.
   - Compute the scalar projection of the data in X onto $\vec{u}$ (a vector with 120 elements).
   - Compute the mean of the vector just computed above.

3. Show that the variance of the projected data (mean subtracted) onto $\vec{u}$ is the same as

   $$(1/(p-1))\mathbf{u}^T \hat{X}^T \hat{X} \mathbf{u}$$

4. Load in the clown data (in Matlab, `load clown`). This gives you a matrix X that is 200×320. To visualize the data,

   ```
   image(X);
   colormap(map);
   ```

   We're going to treat the data as 320 points in R^{200}.

(a) Find the mean vector, and mean subtract the data.

(b) If the mean subtracted matrix is B, we're going to compute some basis vectors using the SVD. We'll explain this later, but for now, type:

```
[U,S,V]=svd(B)
```

The matrix U should be 200×200, and the matrices S and V can be discarded. In fact, we'll only use the first 5 vectors in U:

$$clear S, V U = U(:, 1:5);$$

(c) Now project the data to the basis vectors in U, and plot the first three coordinates in $\mathbb{R}^3$.

(d) Reconstruct the data back into $\mathbb{R}^{200}$, and visualize the result.

Lab Exercises

1. Let A be given as below in Matlab format (so you can copy/paste it):

```
A=[-6 3 -27 -33 -13
 6 -5  25 28 14
 8 -6  34 38 18
12 -10 50 41 23
14 -21 49 29 33];
```

Construct a matrix N whose columns form a basis for the null space of A, and construct a matrix R whose *rows* form a basis for the row space of A. Matlab commands that you may find useful: `rref` and `null`. If you use `null`, use the "rational basis" option so that the vectors are easy to read. In both cases, you should look up the help files- For example, `doc null`.

Perform a matrix computation that confirms the fact that the row space is orthogonal to the null space of a matrix.

2. Let A be given as below in Matlab format. Perform a matrix computation that checks to see if the columns are orthogonal to each other:

```
A=[-6 -3 6 1
-1  2  1 -6
 3  6  3 -2
 6 -3  6 -1
 2 -1  2  3
-3  6  3  2
-2 -1  2 -3
 1  2  1  6];
```

3. Let A be from the previous problem. What is the result of the following Matlab commands?

```
temp=sqrt(sum(A.*A));
B=A./repmat(temp,8,1);
```

4. Let U be the matrix A from the previous question with normalized columns.

   (a) Compare U^TU and UU^T. How do they differ?

   (b) Generate a random vector $\mathbf{y}$ in $\mathbb{R}^8$ and compare

   $$\mathbf{p} = UU^T\mathbf{y} \qquad \text{and} \qquad \mathbf{z} = \mathbf{y} - \mathbf{p}$$

   Explain why $\mathbf{p}$ is in the column space of A. Verify that $\mathbf{z}$ is orthogonal to $\mathbf{p}$.

   (c) Verify that $\mathbf{z}$ is orthogonal to each column of U (one line please!)

   (d) Notice that $\mathbf{y} = \mathbf{p} + \mathbf{z}$, with $\mathbf{p} \in \text{Col}(A)$. Explain why $\mathbf{z} \in (\text{Col}(A))^{\perp}$.

5. Let $\mathbf{y}$ be a random vector in $\mathbb{R}^8$, and let A be the matrix from Exercise 2 (you may also use the matrix U from the previous problem).

   (a) What matrix computations could you use to determine if $A\mathbf{x} = \mathbf{y}$ has a solution?

   (b) If the equation has no solution, what matrix computation would produce the orthogonal projection of $\mathbf{y}$ into the column space of A? (Call it $\mathbf{g}$).

   (c) Now that $A\mathbf{x} = \mathbf{g}$ has a solution, how many solutions does it have? (What matrix computations are you making?)

   (d) Find the solution using the "slash" command. What matrix computations could you use to make sure your solution is in the row space of A?

6. (Optional) Suppose you have a 2×2 matrix A, and you choose the four elements of A at random. What eigenvalues might you expect, and with what distribution? HINT: It depends on whether you use `rand` or `randn` or something else.

   Write Matlab code that will select elements of A at "random", then computes the eigenvalues, and determines if they are real or not. You should output the percentage of time the eigenvalues are real versus complex.