## Topic List

The second exam will be partially in-class, partially take-home. The take-home portion will predominantly be Matlab related.

1. Some Basic Stats

Know how to compute the mean, variance, and correlation by hand and using Matlab. Be able to double-center a matrix (one way is fine, you don't need to know all the alternatives), and mean-subtract data stored as a matrix (using Matlab). Know how to define a discrete probability density function.

Know the definition of the covariance matrix. Be able to prove things about the variance and correlation (like in the exercises in Ch 3 on p. 37).
2. Linear Regression: Be able to find the line of best fit using the normal equations. Be able to set up linear regression problems for other linear models (for example, if the model equation was $y=c_{1} \cos (x)+c_{2} \sin (2 x)$ instead of $\left.y=m x+b\right)$.
Matlab questions: Be able to plot distinct points in the plane. Be able to plot a given function (you have to construct a nice domain and range!).
3. $n$-armed bandit: Be able to discuss the general problem (including supervised versus unsupervised learning), and the details for how we "solved" the problem. In particular, be able to discuss the greedy algorithm, the $\epsilon$-greedy algorithm, the softmax strategy and the pursuit (or win-stay, lose-shift) method. Understand the technique we used to "select action $i$ with probability $P_{i}$ (meaning be able to describe it in words, not using Matlab).
Be able to analyze what happens to our parameters as we iterate the algorithm (for example, bottom of p. 22- find the limits as $\tau \rightarrow 0$ and infinity, and pg. 24 exercises 1-5).
In Matlab: Understand the difference between a script and a function. Be able to write a script for doing the homework (using double percents to separate sections). Understand how to read a Matlab function file to determine how to use it- For example, read the documentation for the Matlab command orth and see what its relationship is with the SVD.
4. Genetic algorithms: Be able to describe what makes up a genetic algorithm (there are four components we identified). Understand how we implemented the four components in the "Binary Strings" problem. Understand how we implemented the four components in the function optimization problem. Finally, understand how we implemented the four components to solve the knapsack problem.
Be able to modify given Matlab code to run the genetic algorithm examples.
5. Basic linear algebra: Be able to define coordinates with respect to a particular basis. Given a basis, be able to compute the coordinates for a given vector. Know what it means to say that two vector spaces are "isomorphic". Be able to talk about the difference between the "high dimensional representation" of a given vector versus its "low dimensional representation".
Define an "orthogonal" matrix. Be able to compute the projection of a vector onto another vector, and onto a subspace.
Be able to compute a projection matrix: For a projection to a vector, For a projection to a subspace spanned by orthonormal columns in a matrix. To project to the column space of a general matrix $A$.
Know the four fundamental subspaces associated to a matrix $A$ (be able to draw a diagram like we did in class). Given the rank of $A$, give the dimensions of all four subspaces. Given a matrix and its rref, be able to construct a basis for the column space, null space and row space.
Show that the null space is orthogonal to the row space. Prove that if $Q$ has orthonormal columns, then $\|Q \mathbf{x}\|=\|\mathbf{x}\|$, and that the dot product between $Q x$ and $Q y$ is the same as the dot product between $x$ and $y$.
Prove the Pythagorean Theorem on two vectors.
6. Eigenvalues/eigenvectors: Recall the three main equations used in eigenvalue/eigenvector computations and proofs. Be able to compute eigenvalues and eigenvectors for various matrices. Know the definition of an eigenspace $\left(E_{\lambda}\right)$, and understand why it is a subspace (therefore, "solving for an eigenvector" is not well defined). Be able to prove some basic facts about eigenvalues. Is there a relationship between eigenvalues and invertibility?
7. The Spectral Theorem: We don't need all of the details here, mainly the following:
"If and $n \times n$ matrix is symmetric, then it has $n$ real eigenvalues and the set of eigenvectors can be formed so that they form an orthonormal basis for $\mathbb{R}^{n}$." Additionally, understand the relationship between the rank and the number of non-zero eigenvalues.
8. The SVD: Be able to state the Singular Value Decomposition, and define "singular values". Be able to compute the SVD by hand for "simple" matrices. Be able to use Matlab to compute the SVD, and from that, be able to find a basis for all four fundamental subspaces. From that, be able to project vectors to any of the four subspaces.
9. The Best Basis: Be able to compute the variance of a set of vectors projected to a single vector, and what relationship this has with the covariance matrix. Find the best basis for a given set of data (using Matlab), and be able to plot the best two dimensional representations.

