Matlab and Linear Algebra, Homework 9A

The following are meant to be done on Matlab (or Octave). Please turn in a printout of your Matlab work, or a listing of your Matlab commands.

1. Let A be given as below in Matlab format (so you can copy/paste it):

   ```
   A=[-6 3 -27 -33 -13
    6 -5  25 28 14
    8 -6  34 38 18
   12 -10 50 41 23
   14 -21 49 29 33];
   ```

   Construct a matrix N whose columns form a basis for the null space of A, and construct a matrix R whose *rows* form a basis for the row space of A. Matlab commands that you may find useful: `rref` and `null`. If you use `null`, use the "rational basis" option so that the vectors are easy to read. In both cases, you should look up the help files- For example, `doc null`.

   Perform a matrix computation that confirms the fact that the row space is orthogonal to the null space of a matrix.

2. Let A be given as below in Matlab format. Perform a matrix computation that checks to see if the columns are orthogonal to each other:

   ```
   A=[-6 -3 6 1
   -1  2  1 -6
    3  6  3 -2
    6 -3  6 -1
    2 -1  2  3
   -3  6  3  2
   -2 -1  2 -3
    1  2  1  6];
   ```

3. Let A be from the previous problem. What is the result of the following Matlab commands?

   ```
   temp=sqrt(sum(A.*A));
   B=A./repmat(temp,8,1);
   ```

4. Let U be the matrix A from the previous question with normalized columns.

   (a) Compare U^TU and UU^T. How do they differ?

(b) Generate a random vector $\mathbf{y}$ in $\mathbb{R}^8$ and compare

$$\mathbf{p} = UU^T\mathbf{y} \qquad \text{and} \qquad \mathbf{z} = \mathbf{y} - \mathbf{p}$$

Explain why $\mathbf{p}$ is in the column space of A. Verify that $\mathbf{z}$ is orthogonal to $\mathbf{p}$.

(c) Verify that $\mathbf{z}$ is orthogonal to each column of U (one line please!)

(d) Notice that $\mathbf{y} = \mathbf{p} + \mathbf{z}$, with $\mathbf{p} \in \text{Col}(A)$. Explain why $\mathbf{z} \in (\text{Col}(A))^{\perp}$.

5. Let $\mathbf{y}$ be a random vector in $\mathbb{R}^8$, and let A be the matrix from Exercise 2 (you may also use the matrix U from the previous problem).

   (a) What matrix computations could you use to determine if $A\mathbf{x} = \mathbf{y}$ has a solution?

   (b) If the equation has no solution, what matrix computation would produce the orthogonal projection of $\mathbf{y}$ into the column space of A? (Call it $\mathbf{g}$).

   (c) Now that $A\mathbf{x} = \mathbf{g}$ has a solution, how many solutions does it have? (What matrix computations are you making?)

   (d) Find the solution using the "slash" command. What matrix computations could you use to make sure your solution is in the row space of A?

6. (Optional) Suppose you have a 2×2 matrix A, and you choose the four elements of A at random. What eigenvalues might you expect, and with what distribution? HINT: It depends on whether you use `rand` or `randn` or something else.

   Write Matlab code that will select elements of A at "random", then computes the eigenvalues, and determines if they are real or not. You should output the percentage of time the eigenvalues are real versus complex.