Week 9 Homework

Matlab Homework

1. Use the Matlab template to solve the knapsack problem. Write down the things you should take:

2. Create 60 random points (or vectors) in $\mathbb{R}^{10}$, and save them in a variable X. Let A be a matrix that is 10×2, and holds 2 orthonormal, non-zero, vectors in $\mathbb{R}^{10}$. We can construct such a matrix using the following commands:

```
[A,R]=qr(randn(10,2),0);
```

   You can discard the matrix R from that. By the way, the QR decomposition is actually the Gram-Schmidt orthogonalization process. Note that the span of the columns of A form a 2-dimensional subspace of $\mathbb{R}^{10}$. Such a subspace is isomorphic to the plane.

   (a) For each point in X, find the coordinates with respect to the columns of A. Hint: The result should be a matrix that is either 60×2 or 2×60, depending on how you set it up.

   (b) Plot the points in the plane (as 60 points in $\mathbb{R}^2$).

   (c) Find the projection of the points in X to the space spanned by the columns of A.

   (d) Find the distance from $\mathbf{x}_1$ to the column space of A.

3. Suppose I have 20 points (or vectors) in $\mathbb{R}^{10}$. If I project all of these points onto a unit vector $\mathbf{u} \in \mathbb{R}^{10}$, all the points will project to scalar multiples of $\mathbf{u}$.

   (a) Show that the mean of the projection is the projection of the mean.

   (b) Assume that the mean is zero, $\sum \mathbf{x}_i = 0$, and recall that the covariance matrix C is defined as:

   $$C = \frac{1}{n-1} \sum_{i=1}^{20} \mathbf{x}_i \mathbf{x}_i^T$$

   Show that the variance of the data projected to $\mathbf{u}$ is given by the following, which is a scalar:

   $$\mathbf{u}^T C \mathbf{u}$$

   Hint: We're taking the variance of the data in the set:

   $$\left\{ \mathbf{x}_1^T \mathbf{u}, \mathbf{x}_2^T \mathbf{u}, \cdots, \mathbf{x}_{20}^T \mathbf{u} \right\}$$

   (c) (Continuing the previous problem) If $\mathbf{v}_1$ happens to be an (unit) eigenvector of C with eigenvalue λ_1, and we project our data to the eigenvector, then what will the covariance be?

4. Try to reason out each of the following Matlab commands. Assume that `x=[1 3 2 1 3]`.

   (a) `x*x'`

   (b) `x.*x`

   (c) `x==max(x)`

5. Write a Matlab script file to plot $sin(x)$ in red, $sin(2x)$ in black, and $sin(3x)$ in green, all in the same plot. Assume $x \in [4, 8]$.