Lab Questions:
Add to our written homework, Due Tuesday

1. This exercise deals with the error function for the formulation of what it means to have a best basis. First, load the data from the website online. It will be a matrix X that is 900×2. Visualizing this as data in two dimensions, it will be elliptical in shape.

   (a) Write a Matlab function that will input a $m \times 2$ matrix, a 2×1 vector and will output the "Error" in using that vector to represent the m data points. The first line of the function will be:

   ```
   function y=ProjError(X,u)
   ```

   To use this function, if matrix X and vector u have been defined, type:

   ```
   y=ProjError(X,u)
   ```

   (b) Let the vector **u** be defined as a unit vector coming away from the origin. We note that any such vector can be defined by using one parameter, θ, where θ is the angle the vector makes with the positive "$x-$axis".

   In Matlab, how do we:

   - Define a vector θ that goes from 0 to 2π using 200 values.
   - Construct a matrix of column vectors u with the corresponding angles from θ (the matrix U will then be 200×2 or 2×200- Your choice)

   (c) Find the 200 values of the error function (using the 200 vectors u constructed in the previous question) and plot the results. Find the vector corresponding to the minimum (graphically or numerically).

   (d) Compare your previous answer with the vector that actually gives you the minimum error.

   (e) Project the data to the best one dimensional subspace. Verify numerically that the new one-dimensional dataset has zero mean, and compute its variance.

2. Here is a complete experiment using the data from the clown image. Type the following as a script file, and answer the questions in the comments:

   ```
   load clown
   figure(1)
   imagesc(X)          % imagesc is short for image scale
   colormap(summer)    % Give 3 other built-in colormaps
   colorbar            % What does this command do?

   [m,n]=size(X);      % What does this command do?
   Xmean=mean(X);
   Xhat=X-repmat(Xmean,m,1);
   ```

```
[U,S,V]=svd(Xhat,'econ');  % Why are we using "econ"?

%The following gives a six dimensional reconstruction of the image:
H=U(:,1:6)*S(1:6,1:6)*V(:,1:6)'+repmat(Xmean,m,1);

figure(2)               %What does this command do?
imagesc(H)
colormap(gray)

Coords=Xhat*V(:,1:3);    % Are these coordinates?  In what sense?
figure(3)
plot3(Coords(:,1),Coords(:,2),Coords(:,3),'.')
Recon=Coords*V(:,1:3)'+repmat(Xmean,m,1);
figure(4)
imagesc(Recon);
```