Matlab for the Best Basis

Miscellaneous Image Commands

We will be treating an image that has $m \times n$ pixels as a vector in $\mathbb{R}^{mn}$. Matlab has some easy commands to do conversions and visualizations. Let A be an image in $\mathbb{R}^{m \times n}$:

- `v=A(:);` Converts matrix A to vector $\mathbf{v}$.
- `B=reshape(v,m,n);` Converts $\mathbf{v}$ back to an $m \times n$ array B.
- `imagesc(A)` Short for "image scale", this results in an $m \times n$ array of colors, where each color is from the value in A. It is a good idea to use `imagesc` rather than the non-scaled version, `image`, unless you know what you're doing.

  You might try the following example, which also shows you how to use some built-in colorings. The colorbar command is useful to give you a color legend.

  ```
  load clown
  imagesc(X)
  colormap(summer)
  colorbar
  ```

  Type `doc colormap` to see all of the built-in options.

Some data plots

If you have an array of data, sometimes it is useful to plot certain coordinates as one type, and other coordinates as another type. For example, try typing the following. The first two lines just build a data set in a matrix X:

```
X1=rand(100,2); X2=rand(100,2)+4;
X=[X1;X2];
index01=1:100;
index02=101:200;
plot(X(index01,1),X(index01,2),'r.',X(index02,1),X(index02,2),'b^');
```

The SVD in Matlab

Given a matrix X, be sure to mean subtract before taking the SVD. Here are the command to produce the "economy" size SVD. For example, here we take the SVD of the clown, and we will assume that the image (which is 200×320)

```
clear           %Get rid of old variables
load clown
[m,n]=size(X);  %Get the dimensions

figure(1)       %Use multiple figure windows
image(X);       %The original clown image
colormap(map);  %This was stored with the clown.

Xmean=mean(X);
Xhat=X-repmat(Xmean,m,1);
[U,S,V]=svd(Xhat,'econ');
```

Now do a 6-dimensional reconstruction of the image, and visualize it:

```
H=U(:,1:6)*S(1:6,1:6)*V(:,1:6)'+repmat(Xmean,m,1);

figure(2)        %Second figure
imagesc(H)
colormap(gray)
```

The Best Basis in Matlab

The best basis is given by the first k eigenvectors of U or V (depends on whether your data is in columns or rows). Assuming the SVD has been computed, then here are some common tasks, where we assume that X is $p \times n$ (to match the text), with p points in $\mathbb{R}^n$.

- The low dimensional representation of the data using the first k columns of V is:

  ```
  Coords=X*V(:,1:k)
  ```

  Note that the dimensions match up appropriately. To get these data points represented back in $\mathbb{R}^n$,

  ```
  Recon=Coords*V(:,1:k)'
  ```

  This is the same as: `X*V(:,1:k)*V(:,1:k)'`, which shows that it is the orthogonal projection of X into the space spanned by the first k columns of V. Be sure to add the mean back in, if it was subtracted out (like we did with the clown image).

As an example, continuing with the clown image, if we view the matrix as 200 points in $\mathbb{R}^{320}$, plot the best three dimensional representation, then show the three dimensional reconstruction back as an image:

```
Coords=Xhat*V(:,1:3);

figure(3)      %Third figure window
plot3(Coords(:,1),Coords(:,2),Coords(:,3),'.')
Recon=Coords*V(:,1:3)'+repmat(Xmean,m,1);

figure(4)      %Fourth figure window
imagesc(Recon);
```

Programming note: We could have put all four graphs in the SAME figure. To do this, replace the lines shown in the table. The only problem with this is that you are only allowed one colormap per figure (You'll see what happens):

Replace:	With:
`figure(1)`	`subplot(2,2,1)`
`figure(2)`	`subplot(2,2,2)`
`figure(3)`	`subplot(2,2,3)`
`figure(4)`	`subplot(2,2,4)`

To Prepare for the Quiz on Friday

I will be giving you a matrix X that will be $77,028 \times 30$, representing 30 points in $\mathbb{R}^{77028}$, where each comes from a 294×262 array.
Think about how to do the following in Matlab:

- Find the mean (in $\mathbb{R}^{77028}$) and mean-subtract the data. Visualize the mean as a 294×262 array (using `reshape` and `imagesc`).
- Find the best basis in $\mathbb{R}^{77028}$, and visualize the basis vectors as 294×262 arrays (four graphs in one figure- Look up `subplot`)
- Plot the best two dimensional representation of the data.
- Show a 2, 5 and 15-dimensional reconstruction of a random selection of 4 data points (or arrays), and visualize them in a subplot.