HW Solutions from Mar 10

Section 2.4, p. 22: #2 (computer), 6, 7 (#1-7 assigned, but turn in only those three).

2. (a) I wanted you to see that $\sin(t)$ and $\cos(t)$ were uncorrelated, but the question left out an important point: We should use equally spaced points between 0 and 2π for this to work. Matlab code and output:

```
>> t=linspace(0,2*pi)';
>> v=sin(t); u=cos(t); u'*v

ans =

  -1.8180e-15
```

(b) For the correlation, Matlab gives a vector. The off-diagonal entries are the correlation coefficient.

```
>> corrcoef(u.^2,v.^2)
ans =
     1    -1
    -1     1
```

(c) Same idea as the previous problem.

```
>> y=2*t-5; corrcoef(t,y)
ans =
     1     1
     1     1
>> y=-t+5; corrcoef(t,y)
ans =
    1.0000   -1.0000
   -1.0000    1.0000
```

6. Let s_x^2 be the variance of $\boldsymbol{x}$, so that $s_x^2 = \frac{1}{p-1}\sum_{k=1}^{p}(x-\bar{x})^2$ Now, the covariance of $\mathbf{x}$ and $a\mathbf{x}+b$ is given by the following:

$$\frac{1}{p-1}\sum_{k=1}^{p}(x-\bar{x})(ax-b-\overline{ax-b})$$

We found that $\overline{ax-b} = a\bar{x}-b$ in a previous problem, so

$$(ax-b)-(\overline{ax-b}) = (ax-b)-(a\bar{x}-b) = a(x-\bar{x})$$

Substituting this back in, we get:

$$\frac{1}{p-1}\sum_{k=1}^{p}(x-\bar{x})a(x-\bar{x}) = as_x^2$$

7. For the correlation coefficient,

$$r_{xy}^2 = \frac{s_{xy}^2}{s_x s_y}$$

Recall that we just showed: $s_{xy}^2 = as_x^2$, and in Problem #4, $s_y^2 = a^2 s_x^2$, so substituting these values in, we get

$$r_{xy}^2 = \frac{s_{xy}^2}{s_x s_y} = \frac{as_x^2}{s_x\sqrt{(a^2 s_x^2)}} = \frac{as_x^2}{s_x|a|s_x} = \frac{a}{|a|}$$

This expression turns out to be the "sign" (aka "signum") of a: +1 if $a > 0$, -1 if $a < 0$, and we'll assume that $a \neq 0$.