Overview and Summary, Chapters 1 and 2

Exam 1 will cover topics from Chapters 1 and 2. Some formulas will be provided if needed- Those were the four cosine and sine sum formulas, and the formulas for the heat equation in polar coordinates (like at the end of section 1.5 and 2.5). You may use the following results without proof: For $m \neq n$,

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right)\, dx = 0 \qquad \int_0^L \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{m\pi x}{L}\right)\, dx = 0$$

And for $n = m$, the integrals become:

$$\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)\, dx = \frac{L}{2} \qquad \int_0^L \cos^2\left(\frac{n\pi x}{L}\right)\, dx = \frac{L}{2}$$

You can use similar, appropriate integrals for the interval $[-L, L]$, as we did for the ring.

As for differential equations, be able to solve the general first order linear differential equation:

$$y' + a(t)y = p(t)$$

and the second order linear differential equation in all three cases:

$$ay'' + by' + cy = 0$$

We also talked about the Euler DE. To get the characteristic equation, use $y = t^r$ for the ansatz. To remember the three cases, remember that this equation is equivalent to the regular second order DE if we use the substitution $x = \ln(t)$.

$$t^2y'' + \beta ty' + \gamma y = 0$$

Recall the definition of $\cosh(x)$ and $sinh(x)$. We'll be using these again and again, so it's good to be familiar with them.

General Vocabulary

- PDE; Solution to a PDE
- Linear operator, linear PDE, homogeneous PDE. Linear/Homogeneous boundary conditions. Superposition principle.
- Boundary conditions, initial conditions, rule of thumb for number of conditions needed.
- Equilibrium solution (or "steady state")
- Newton's law of cooling (in words; write for boundary condition, like equation 1.3.4, 1.3.5, and in the summary on that page)
- Gradient, Laplacian.
- Eigenvalue and eigenfunction for a linear operator.

Concepts, Definitions and "Tricks"

1. When discussing heat, we introduced: $e(x,t)$ is the thermal energy density, $\phi(x,t)$ is the flux and $u(x,t)$ is the temperature. We also had $Q(x,t)$ to represent internal heat sources (as a density function).

2. From the Fundamental Theorem of Calculus,

   $$\phi(a,t) - \phi(b,t) = -\int_a^b \phi_x(x,t)\,dx$$

3. Fourier's Law of Diffusion $\phi = -K_0 u_x$

4. What does Conservation of Energy say in the context of heat in a rod? (This was the first thing we had in modeling the heat equation).

5. Relationship between thermal energy density and temperature:

   $$e(x,t) = c\,\rho\,u(x,t)$$

   where c, ρ could possibly be functions of x. They represent the specific heat and mass density, respectively. I won't ask you to define c, ρ (but do remember that they could be functions of x).

6. Total thermal energy in the rod:

   $$\int_0^L e(x,t)\,A\,dx = Ac\rho \int_0^L u(x,t)\,dx$$

7. To see if an equilibrium solution exists, we can see if the total energy in the rod changes. Or, find the derivative of the total thermal energy (show that it is zero).

   $$\frac{d}{dt}\int_0^L u(x,t)\,dx = 0 \quad \Rightarrow \quad \int_0^L u_t(x,t)\,dx = 0 \quad \Rightarrow \quad \int_0^L (ku_{xx}(x,t) + Q(x,t))\,dx = 0$$

   Continuing with this computation, we might note:

   $$k(u_x(b,t) - u_x(a,t)) + \int_0^L Q(x,t)\,dx = 0$$

8. A set of functions, $\{\phi_1(x), \phi_2(x), \ldots, \}$ is said to be **orthogonal on the interval** $[a,b]$ if

   $$\int_a^b \phi_i(x)\phi_j(x)\,dx = \left\{ \begin{array}{cc} 0 & \text{if } i \neq j \\ \neq 0 & \text{if } i = j \end{array} \right\}$$