Review: Exam 2

Exam 2 covers material from Chapters 3, 4 and 5:

1. Chapter 3:

   Chapter 3 formalizes the things we did in Chapter 2 when using separation of variables. That is, we looked at the Fourier series:

   $$a_0 + \sum_{n=1}^{\infty} a_n \cos(\sqrt{\lambda_n}x), \qquad \sum_{n=1}^{\infty} b_n \sin(\sqrt{\lambda_n}x)$$

   on the half interval $[0, L]$ and the full Fourier expansion on $[-L, L]$:

   $$a_0 + \sum_{n=1}^{\infty} a_n \cos(\sqrt{\lambda_n}x) + \sum_{n=1}^{\infty} b_n \sin(\sqrt{\lambda_n}x)$$

   We looked at when these series converged, and to what they converged. We also looked at when it was valid to differentiate/integrate the Fourier series term by term.

   Details:

   - Section 3.1:
     - Define PWS, the periodic extension of f
   - Section 3.2:
     - Definition of the Fourier Series (on $[-L, L]$), with definitions of the coefficients.
     - The difference between the function $f(x)$ and the Fourier series for f (note the $\sim$)
     - The Convergence Theorem (Fourier's Theorem, or the Fundamental Theorem for Fourier Series)
     - Note that $\cos(n\pi) = (-1)^n$ when computing Fourier coefficients.
   - Section 3.3:

     In Section 3.3, we focus on the differences between the full Fourier series of 3.2, and look at the sine series and cosine series on the half interval $[0, L]$.

     Understand the Gibbs phenomena.

     Key idea: Be sure to understand the difference between the even and odd *extension* of f, and the even and odd *parts* of f.
   - Section 3.4:

     When can we perform term-by-term differentiation of the Fourier series, and get something that represents the derivative (overall)? In particular, understand the derivation of 3.4.13, p. 117.
   - Term-by-Term integration: (Summary of the theorem is all we need)
   - The complex form.

   From Chapter 3 overall:

   (a) Definitions:

   PWS, jump discontinuity, periodic extension, even/odd functions, the even/odd extension of f, then even/odd part of f,

   (b) Theorems:

   The Convergence Theorem (Fundamental Theorem of Fourier Series). The 3 convergence theorems at the end of 3.3 are a nice summary for the convergence of the cosine/sine series. Know when a series can be differentiated term by term. It's a good idea to understand the proof on 116-117.

   For integration, just know that if f is PWS, then the series can be integrated term by term.

(c) Computation:
Full Fourier series, Fourier coefficients on $[-L, L]$, Fourier sine or cosine series (on the half interval $[0, L]$, and the coefficients).

(d) Other:
Understand the Gibbs phenomenon. Be able to use the Method of Eigenfunctions (p. 118).
We won't do much with the complex form, but you should understand what the notation is:

$$f(x) \sim \sum_{n=-\infty}^{\infty} c_n \mathrm{e}^{-in\pi x/L}$$

2. Chapter 4:

Chapter 4 is much like a repetition of Chapter 1, except for the wave equation. The key now is that we use separation of variables, but we can also bring our understanding of the Fourier series to bear on this new problem.

In particular, we looked at how to solve the wave equation under various boundary conditions. For Chapter 4 overall,

(a) Computation: Use separation of variables to solve the wave equation with various BCs. Understand what the normal modes of vibration are, and the circular frequency.

(b) Other:
For the modeling, know what $\rho, T, Q(x, t)$ represent, and the general form of the (vertical) motion is:

$$\rho_0(x) u_{tt} = \frac{\partial}{\partial x}(T(x, t) u_x) + \rho_0(x) Q(x, t)$$

so that the simplification becomes: $u_{tt} = c^2 u_{xx}$.

3. Chapter 5:

Chapter 5 is again a little more theoretical, and leads us on an investigation as to whether or not the nice things about the Fourier series (items from the "mega-theorem") will generalize.

As it turns out, the key to getting the generalization is that the ODE (from separation of variables applied to a given PDE) is a Sturm-Liouville operator, and we have regular boundary conditions.

- Section 5.2: Examples of the form of the PDE in a couple of different cases.
- Section 5.3:
  We defined the Sturm-Liouville eigenvalue problem, with regular BCs. We stated the "Mega-Theorem". You don't need to repeat the theorem, but do understand what it says. For example, the theorem says that eigenfunctions corresponding to different eigenvalues are orthogonal with respect to $\sigma(x)$.
  We also defined the Rayleigh quotient, and showed how to get it and what it means (in terms of the eigenvalues).
- Section 5.4: A worked example, showing how to use the "mega-theorem" in the abstract.
- Section 5.5: Self-adjoint operators and proofs of a couple of the statements in the mega-theorem. In particular, we got Lagrange's identity and Green's formula for working with the expression $\int uL(v) - vL(u)\, dx$
  NOTE: If you've had linear algebra, you should work through the appendix to give yourself a better understanding of 5.5, however, this is optional and will not be on the examination.
- Section 5.6: A nice focus on the Rayleigh quotient and its usefulness.
  Key: The minimization principle, use of trial functions,
- Section 5.7: A worked example illustrating the ideas of 5.6.