Homework Set 1

1. Partial differential equations can be classified many ways. A second order **linear** PDE in two variables is any function $u(x, y)$ whose PDE can be expressed as:

$$Au_{xx} + Bu_{xy} + CU_{yy} + Du_x + Eu_y + Fu = G$$

where A, B, C, D, E, F, G can be constants or functions of x, y. For example,

$$u_{tt} = \mathrm{e}^{-t} u_x + \sin(t)$$

is a linear second order DE. Furthermore, if $G = 0$, the equation is called **homogeneous**. For each equation below, state the order of the DE, state whether the PDE is linear or nonlinear, and whether it is homogeneous or not homogeneous:

   (a) $u_t = u_x^2 + 2u_x + u$

   (b) $u_t = u_{xx} + 2u_x + u$

   (c) $u_{tt} = uu_{xxxx} + \mathrm{e}^{-t}$

2. How many solutions can you find to the PDE $u_t = u_{xx}$? You might try a couple different approaches: (i) Assume u is a function of one variable only, and (ii) Assume $u = \mathrm{e}^{ax+bt}$.

3. If $u_1(x, y)$ and $u_2(x, y)$ each satisfy the DE:

$$Au_{xx} + Bu_{xy} + CU_{yy} + Du_x + Eu_y + Fu = G$$

Then is it true that the sum satisfies it as well?

4. **Review Question, Calc 2**: Compute the following integrals

$$\int \cos(t)\,dt \qquad \int \cos^2(t)\,dt \qquad \int \cos^3(t)\,dt \qquad \int \cos(t)\sin(t)\,dt$$

5. **Review Question, Calc 3**: Recall that the line integral can be expressed a couple of different ways, depending on how we set things up. For example, here are two ways of defining the same thing:
If $f : \mathbb{R}^2 \to \mathbb{R}$, and the curve C is defined using the parametric function $\mathbf{r}(t)$, then

$$\int_C f(\mathbf{r}(t))\,ds = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|\,dt$$

Exercise: Evaluate $\int_C x^2 + xy\,ds$, where C is the upper half of the unit circle.

6. (*)**Review Question, Calc 3**: If $\mathbf{F}$ is a vector field on a smooth curve C defined by $\mathbf{r}(t)$, then the work done by $\mathbf{F}$ in moving a particle along the curve C is given by

$$W = \int_C \mathbf{F} \cdot d\mathbf{r}$$

Exercise: Find the work done using

$$\mathbf{F} = \langle x, y, xy\rangle \qquad \mathbf{r}(t) = \langle \cos(t), \sin(t), t\rangle \qquad 0 \le t \le \pi$$

7. From Section 1.2:

   (a) (*) 1.1.1(a, b)

   (b) 1.2.2: As a “hint”, start with Equation 1.2.4, and follow that derivation.

   (c) 8