Homework: Chapter 8

1. For each of the following problems, find an appropriate function w that satisfies the boundary conditions, then let $u = v + w$ and convert the PDE to a PDE in v. Do not solve the PDE.

   (a)
   $$\begin{array}{ll} \text{PDE} & u_t = ku_{xx} + x \quad 0 < x < L \\ \text{BCs} & u_x(0,t) = 1, \quad u(L,t) = t \end{array}$$

   (b)
   $$\begin{array}{ll} \text{PDE} & u_t = ku_{xx} + x \quad 0 < x < L \\ \text{BCs} & u_x(0,t) = t, \quad u_x(L,t) = t^2 \end{array}$$

   (c)
   $$\begin{array}{ll} \text{PDE} & u_{tt} = c^2 u_{xx} + xt \quad 0 < x < L \\ \text{BCs} & u(0,t) = 1, \quad u(L,t) = t \end{array}$$

   (d)
   $$\begin{array}{ll} \text{PDE} & u_{tt} = c^2 u_{xx} + xt \quad 0 < x < L \\ \text{BCs} & u_x(0,t) = 0, \quad u_x(L,t) = 1 \end{array}$$

2. Solve 1(a) and (b) if the initial condition is $u(x,0) = f(x)$.

3. Solve:
   $$\begin{array}{ll} \text{PDE} & u_t = ku_{xx} + \mathrm{e}^{-t} \quad 0 < x < \pi \\ \text{BCs} & u_x(0,t) = 0, \quad u_x(\pi,t) = 0 \\ \text{IC} & u(x,0) = \cos(2x) \end{array}$$