Math 367 Notes and Homework Set 2

A Useful Lemma

If $f(x)$ is continuous, and $\displaystyle\int_a^b f(x)\,dx = 0$ for arbitrary $a < b$, then $f(x) = 0$.

Some notes about this:

- To prove the Lemma, we would need to show that, if c is some point in the interval $[a, b]$, and $f(c) > 0$, then there exists an interval $(c - \epsilon, c + \epsilon)$ on which $f(x) > 0$. In that case, $\int_{c-\epsilon}^{c+\epsilon} f(x)\,dx > 0$, which is a contradiction.
- If f is not continuous, we can not make this statement- For example, consider a function like:

$$f(x) = \begin{cases} 1 & \text{if } x = 1, 1/2, 1/3, 1/4 \\ 0 & \text{otherwise} \end{cases}$$

  then f is not identically zero, but $\int_a^b f\,dx = 0$.
- This lemma is also not the same as the one dealing with odd functions- That was, if f is an odd function, then by symmetry, $\int_{-a}^{a} f(x)\,dx = 0$

Homework Day 2

1. (*) Let n be any positive constant. Show directly that any function of the form

$$u(x, t) = \mathrm{e}^{-(n\pi\alpha)^2\,t} \sin(n\pi x)$$

   solves the heat equation, $u_t = \alpha^2 u_{xx}$.

2. (*) A quick look at units: The units we use for temperature $u(x, t)$ is typically degrees Celsius, and suppose that time is in seconds, and x is in centimeters. What are the units of the thermal diffusivity constant k so that the units all match up in the heat equation,

$$u_t = k u_{xx}$$

3. Text 1.3.3(*)

   Hint: Use e, ϕ, u as usual. The conservation law for the energy of the bath can be thought of as: If $E(t)$ is the thermal energy density of the bath, then the rate of change of the total energy is equal to the energy out of the end of the bar, or (can you figure out what V and A must be?):

$$\frac{d}{dt}(E(t)V) = \phi(L, t)A$$