Homework, 3.1-3.2

Starred problems are due next Wednesday, Feb 27th.

1. **Section 3.2:** 1(a,b,e,f), 2(a,b,g*), 4

2. Practice with the definitions:

   (a) Show that f is continuous, but not piecewise continuous and not piecewise smooth:

   $$f(x) = 1/x^2 \qquad \text{on } (0,1)$$

   (b) Show that f is not continuous, but is piecewise continuous and piecewise smooth.

   $$f(x) = \begin{cases} x^2 & \text{if } -\pi < x < 0 \\ x^2+1 & \text{if } 0 \le x < \pi \end{cases}$$

   (c) (*) In this case, show that f is continuous on the interval and $f'(x)$ exists for every x in the interval, but f is not piecewise smooth (focus on the origin).

   $$f(x) = \begin{cases} x^2\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \qquad \text{on the interval } -\pi \le x \le \pi$$

   (d) If f, g are piecewise smooth, show that $f+g$ and fg are both piecewise smooth. (Hint: Think about limit laws).

3. Properties of even and odd functions. Answer (by proving your statement):

   (a) The product of an even function times an even function is?

   (b) The product of an even function times an odd function is?

   (c) The product of an odd function times an odd function is?

   (d) What is $\int_{-L}^{L} f(x)\,dx$, if f is odd?

   (e) Show that $\int_{-L}^{L} f(x)\,dx = 2\int_0^L f(x)\,dx$, if f is even:

4. Making a function even and odd: Suppose $f(x)$ is defined on the interval $[0, L]$. Then we can make f odd on the interval $[-L, L]$ by defining g:

   $$g(x) = \begin{cases} -f(-x) & \text{if } -L \le x < 0 \\ f(x) & \text{if } 0 \le x \le L \end{cases}$$

   Find a similar formula for making f even on $[-L, L]$ if f is originally defined on $[0, L]$.

5. Integration practice:

$$\int \sin^2(n\pi x)\,dx = ? \qquad \int x\sin(n\pi x)\,dx = ? \qquad \int \mathrm{e}^{-2x}\sin(n\pi x)\,dx = ?$$

6. We said that if f is defined on $[a, b)$, then the periodic extension of f is given by:

$$f\left(x - \left\lfloor \frac{x-a}{b-a} \right\rfloor (b-a)\right)$$

For example, if $f(x) = x^2$ on $[-1, 3)$, what is $f(7)$? What is $f(-2)$? Check your answer graphically, and sketch the periodic extension of f. In fact, try it using Maple:

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f:=x->x^2; a:=-1; b:=3;
plot(f(x-floor( (x-a)/(b-a) )*(b-a)),x=-3..10,discont=true);
```

We said in class that we're partitioning the real line into intervals:

$k = 0$	Interval: $[a, b)$
$k = 1$	Interval: $[a + (b-a), a + 2(b-a))$
$k = 2$	Interval: $[a + 2(b-a), a + 3(b-a))$
$\vdots$	$\vdots$

where $k = \lfloor (x-a)/(b-a) \rfloor$. Test this with $a = -1$, $b = 3$ and $x = 2$, then compute which interval $x = 19$ is in.