Homework, Sections 3.4-3.5

1. **From the text**: 3.4: 6, 9, 12(*), 13.

   Hint on 13: Let $u(x,t) = \phi(x,t) + v(x,t)$ where ϕ is the simplest function that will satisfy the boundary conditions:

   $$\phi(0,t) = A(t) \qquad \phi(L,t) = 0$$

   Next, we should be able to build the PDE in v so that it has homogeneous boundary conditions.

2. **From the text:** 3.5: 1, 3.6: 1

3. (Practice with integration by parts) Consider the integral:

   $$\int_a^b f''(x)\, e^{-kx}\, dx$$

   Rewrite this using integration by parts so that the integral only uses $f(x)$ itself.

4. Solve the following PDE using the method of eigenfunctions:

   $$
   \begin{array}{ll}
   \text{PDE} & u_t = u_{xx} + \sin(\pi x), \quad 0 < x < 1 \\
   \text{BCs} & u(0,t) = 0 \quad u(1,t) = 0 \\
   \text{ICs} & u(x,0) = 1
   \end{array}
   $$