```
> # Heat Equation, Insulated BCs
  # (Thanks to James Herod's website)

> #Initializations:
  restart;
> with(plots):
> with(PDEtools):
> # The PDE
  u:=(t,x)->X(x)*T(t);
```

$$u := (t, x) \rightarrow X(x)\ T(t) \tag{1}$$

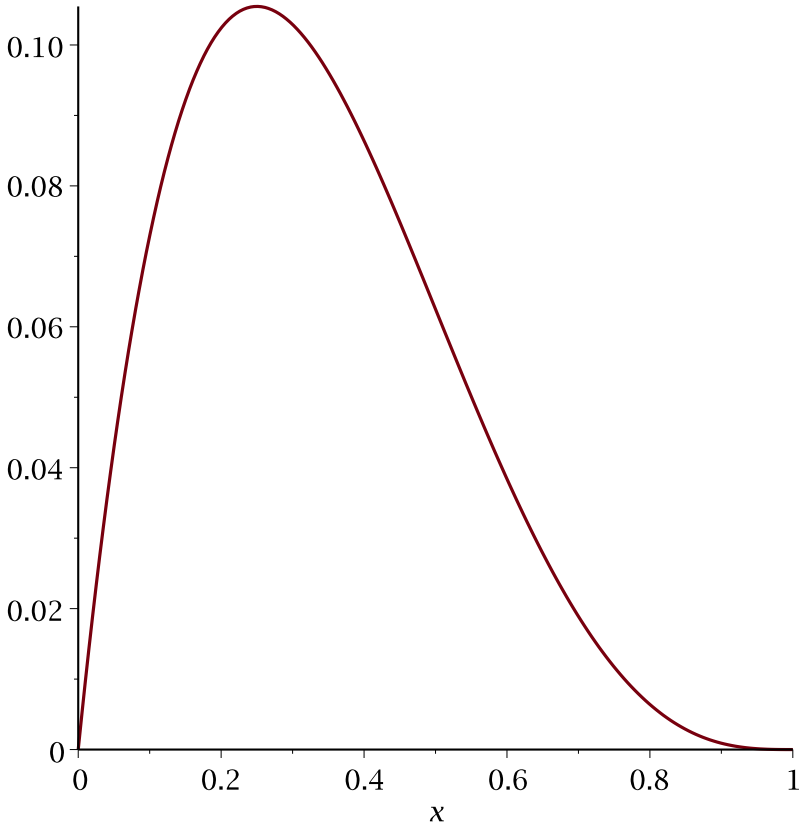
```
> diff(u(t,x),t)=diff(u(t,x),x,x);
```

$$X(x)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ T(t)\right) = \left(\frac{\mathrm{d}^2}{\mathrm{d}x^2}\ X(x)\right) T(t) \tag{2}$$

```
> #IC:
> f:=x->x*(1-x)^3;
```

$$f := x \rightarrow x\,(1-x)^3 \tag{3}$$

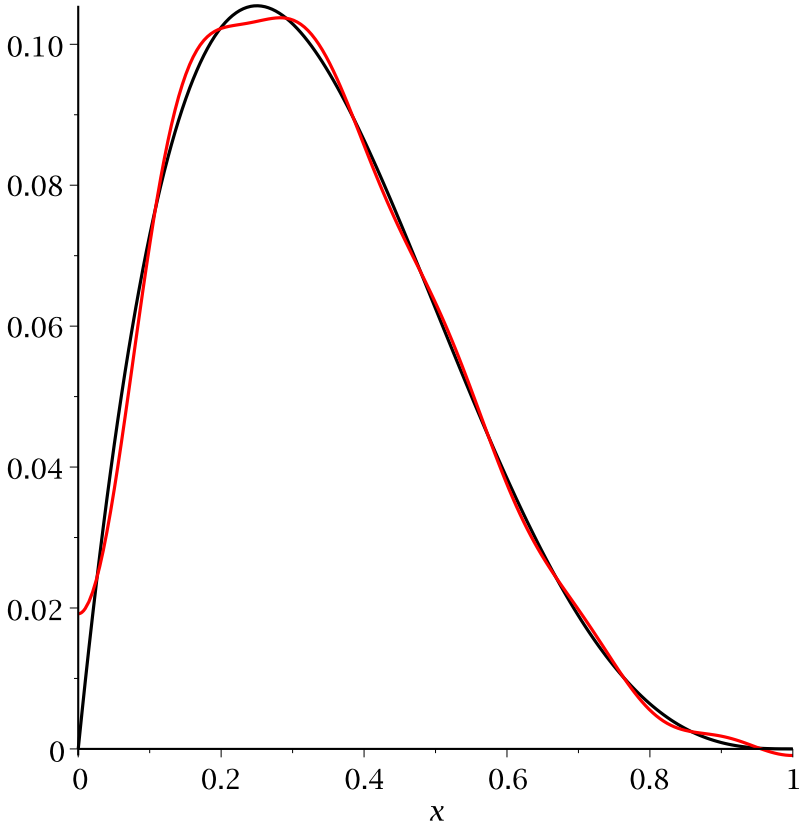
```
> plot(f(x),x=0..1);
```



```
> # Build a partial sum with 10 components:
> for n from 0 to 10 do
    c[n]:=int(f(x)*cos(n*Pi*x),x=0..1)/int(cos(n*Pi*x)^2,x=0..1):
  od:
  n:='n';
```

$$n := n \tag{4}$$

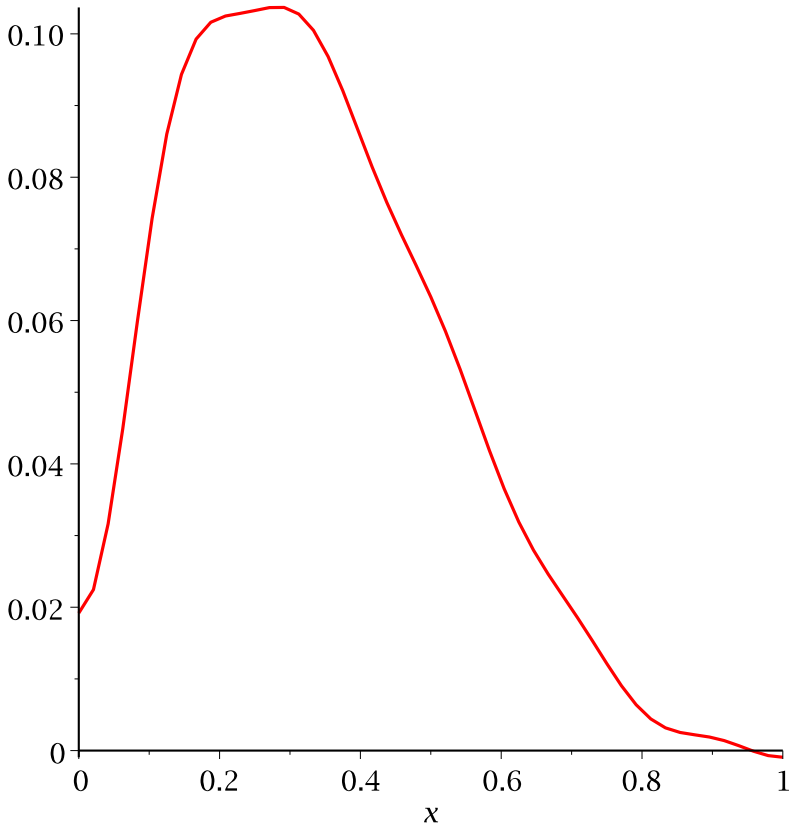
```
> approx:=x->sum(c[n]*cos(n*Pi*x),n=0..10):
> plot([f(x),approx(x)],x=0..1,color=[BLACK,RED]);
```



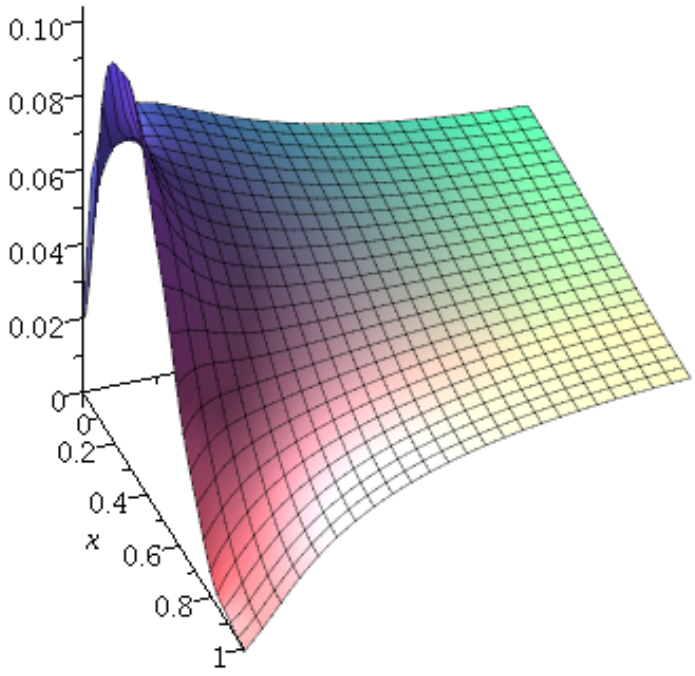

```
> u:=(t,x)->sum(c[n]*exp(-n^2*Pi^2*t)*cos(n*Pi*x),n=0..10);
```

$$u := (t, x) \rightarrow \sum_{n=0}^{10} c_n \, e^{-n^2 \pi^2 t} \cos(n \pi x) \tag{5}$$

```
> # Animate the curve through time (click on the plot, then press
  the play key)
> animate(u(t,x),x=0..1,t=0..1/3);
```




```
> # Plot the temperature over position and time in 3-d:
> plot3d(u(t,x),x=0..1,t=0..0.3,axes=NORMAL,orientation=[-20,55]);;
```




```
> #Check that the energy indeed has not changed over time:
  int(f(x),x=0..1);
```

$$\frac{1}{20} \tag{6}$$

```
> c[0];
```

$$\frac{1}{20} \tag{7}$$