Exercise Set 1 to Replace 4.2-4.4

1. Let $f(w) = \mathrm{e}^{-w}$. Show by direct computation that a solution to the wave equation, $u_{tt} = c^2 u_{xx}$, can be written as the sum of a right and left traveling wave,
$$u(x,t) = f(x-ct) + f(x+ct)$$

2. (4.2.1) Consider the wave equation,
$$u_{tt} = c^2 u_{xx} - g$$
Find the equilibrium solution, $u_e(x)$, if $u_e(0) = u_e(L) = 0$.

3. (Continuing from the previous problem) Show that $v(x,t) = u(x,t) - u_e(x)$ satisfies the homogeneous wave equation, $u_{tt} = c^2 u_{xx}$.

4. Solve the wave equation (using separation of variables):
$$\begin{aligned} u_{tt} &= 4u_{xx} \qquad 0 < x < \pi \\ u(0,t) = 0 &\quad u(\pi,t) = 0 \\ u(x,0) &= \sin(x) \\ u_t(x,0) &= 0 \end{aligned}$$

5. Solve the wave equation (using separation of variables):
$$\begin{aligned} u_{tt} &= 25u_{xx} \qquad 0 < x < 2 \\ u(0,t) = 0 &\quad u(2,t) = 0 \\ u(x,0) &= 0 \\ u_t(x,0) &= \sin(\pi x) + \tfrac{1}{10}\sin(2\pi x) \end{aligned}$$

6. Use Maple or Mathematica to show your solutions to #4 and 5. Show either an animation (animated in time) or a three dimensional plot of the solution, $u(x,t)$.

7. Solve the wave equation (using separation of variables):
$$\begin{aligned} u_{tt} &= 4u_{xx} - ku_t \qquad 0 < x < 1, k > 0 \\ u(0,t) = 0 &\quad u(1,t) = 0 \\ u(x,0) &= f(x) \\ u_t(x,0) &= 0 \end{aligned}$$
For this problem, assume further that k is a very small number. Any coefficients in your solution should be expressed as appropriate integrals.