Topics for Review

A few formulas will be provided (sample will be attached to the review). No calculators or other notes will be allowed. The exam will be approximately an exam and a half in length, but we will have about 2 hours to complete it. The exam will be about 2/3 over the new material, 1/3 over the older material, but you should note that we are reviewing the older material in the newer sections, so it's mixed together quite a bit. I would recommend going over the homework problems from the most recent section, then go over the exam reviews and old exams for the earlier material. If you have time, you might peruse some of the older homework as well.

The specific sections we have covered since the second exam are: 5.6-5.8 (The Rayleigh Quotient and numerically determining eigenvalues), 7.1-7.9 (Higher dimensional PDEs and the Helmholtz equation), 8.1-8.3 (Nonhomogeneous PDEs and BCs), and some first order PDEs from Ch 12.

1. 5.6-5.8: We should be able to construct the Rayleigh Quotient from the SL problem. Given a SL problem, use the Rayleigh Quotient to argue that $\lambda \geq 0$. Use trial functions to find an upper bound for λ_1. In Section 5.7, the idea is that we can go ahead and write the solution using eigenfunctions, even if we're not able to numerically write down the eigenvalues. In 5.8, we graphically estimated eigenvalues.

   You should know the general shape of the graph of the tangent, the general shape of the hyperbolic sine and cosine. Other functions would be graphed for you.

2. Ch 7, Sections 1–9: Know how the wave/heat equation generalize to higher dimensions, and how to write the general boundary condition (Equations 7.2.14 and 15). Solve the heat/wave/Laplace's equation over a rectangle in the plane. Solve the heat/wave/Laplace's equation over a circular domain. Know the definition/solutions to Bessel's equation of order m. Know the orthogonality condition for Bessel functions (need extra r term). Know what a nodal curve is and how it is plotted. Think of 7.9 as an extra worked example. Section 7.4 was the main theoretical section- Know which parts of the Sturm-Liouville theory do not translate directly to higher dimensions. You don't need to know the general Helmholtz equation, only the specialized version: $\nabla^2\phi = -\lambda\phi$.

3. Ch 8, Sections 1–3: Solve nonhomogeneous PDEs with or without homogeneous boundary conditions. Here we utilized the method of eigenfunctions.

4. Topics from Ch 12: First order PDEs. Solve using the method of characteristics. The HW sheet will be a good indication of the types of problems to expect.

Miscellaneous Formulas

- $$\frac{1}{L}\int_0^L f(x)\,dx \qquad \frac{1}{2L}\int_{-L}^L f(x)\,dx$$
  $$\frac{2}{L}\int_0^L f(x)\cos(n\pi x/L)\,dx \qquad \frac{1}{L}\int_{-L}^L f(x)\cos(n\pi x/L)\,dx$$
  $$\frac{2}{L}\int_0^L f(x)\sin(n\pi x/L)\,dx \qquad \frac{1}{L}\int_{-L}^L f(x)\sin(n\pi x/L)\,dx$$

- $$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

- $$(p\phi')' + q\phi = -\lambda\sigma\phi \qquad \nabla^2\phi = -\lambda\phi$$

- $$\frac{-p\phi\phi'|_a^b + \int_a^b p(\phi')^2 - q\phi^2\,dx}{\int_a^b \phi^2\sigma\,dx} \qquad \frac{-\oint \phi\nabla\phi\cdot\vec{n}\,ds + \iint_R |\nabla\phi|^2\,dx\,dy}{\iint_R \phi^2\,dx\,dy}$$

- $$\int_a^b uL(v) - vL(u)\,dx = p(uv' - vu')|_a^b$$
  $$\iint_R u\nabla^2 v - v\nabla^2 u\,dx\,dy = \oint (u\nabla v - v\nabla u)\cdot\vec{n}\,ds$$