

```
> with(plots):
> #animate solutions of the wave equation, with free ends
> L:=1; c:=1; NumSol:=40; #NumSol is how many terms of the sum to
include (about 20 is OK)
```

```
L:= 1
```

```
c:= 1
```

```
NumSol:= 40
```

(1)

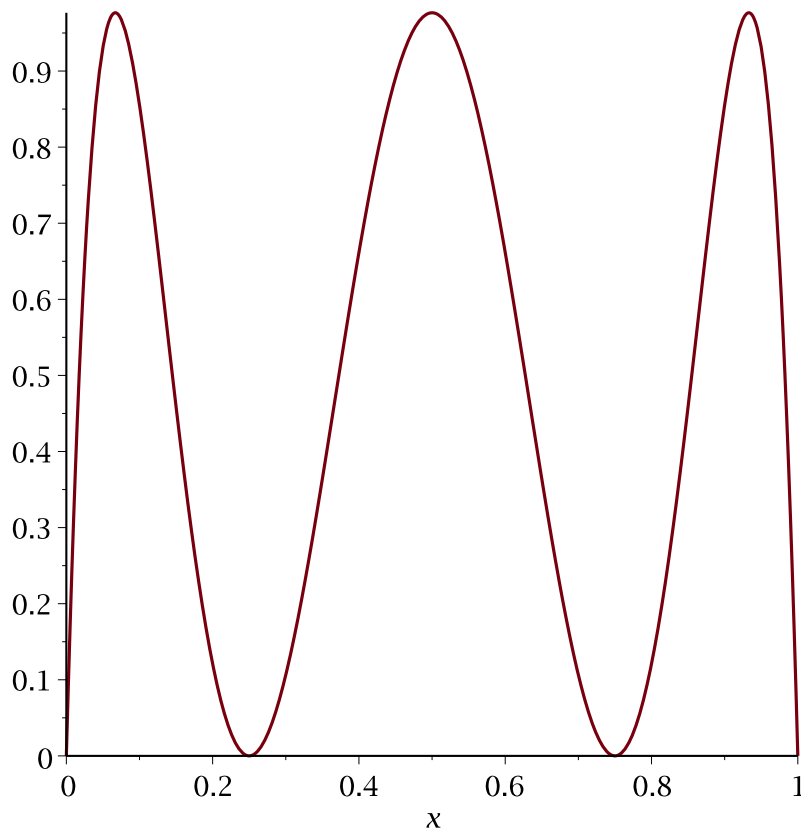
```
> f:=x->-1000*x*(x-1/4)^2*(x-3/4)^2*(x-1);
g:=x->sin(Pi*x);
```

$$f := x \rightarrow -1000 x \left(x - \frac{1}{4}\right)^2 \left(x - \frac{3}{4}\right)^2 (x - 1)$$

$$g := x \rightarrow \sin(\pi x)$$

(2)

```
> plot(f(x),x=0..L);
```



```
> a0:=(1/L)*int(f(x),x=0..L);
a:=n->(2/L)*int(f(x)*cos(n*Pi*x/L),x=0..L);
A:=[seq(evalf(a(n)),n=1..NumSol)];
```

$$a0 := \frac{225}{448}$$

$$a := n \rightarrow \frac{2 \left(\int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx \right)}{L}$$

A := [0., -0.01750258988, 0., 0.4280627300, 0., -0.09519575347, 0.,
-0.1230769119, 0., -0.1008636216, 0., -0.07865472263, 0.,
-0.06167055060, 0., -0.04916751502, 0., -0.03991291451, 0.,
-0.03294931192, 0., -0.02761123088, 0., -0.02344486949, 0.,
-0.02013867138, 0., -0.01747546553, 0., -0.01530117939, 0.,
-0.01350454513, 0., -0.01200382940, 0., -0.01073803191, 0.,
-0.009660952031, 0., -0.008737121565]

```
> b0:=(1/L)*int(g(x),x=0..L);
b:=n->(2/(n*Pi*c))*int(g(x)*cos(n*Pi*x/L),x=0..L);
B:=[seq(evalf(b(n)),n=1..NumSol)];
```

$$b0 := \frac{2}{\pi}$$

$$b := n \rightarrow \frac{2 \left(\int_0^L g(x) \cos\left(\frac{n\pi x}{L}\right) dx \right)}{n\pi c}$$

B := [0., -0.06754745574, 0., -0.006754745574, 0., -0.001929927307, 0.,
-0.0008041363777, 0., -0.0004093785196, 0., -0.0002361799152, 0.,
-0.0001484559466, 0., -0.00009933449373, 0., -0.00006970841665, 0.,
-0.00005078756070, 0., -0.00003814085586, 0., -0.00002936845902, 0.,
-0.00002309314726, 0., -0.00001848589374, 0., -0.00001502724265, 0.,
-0.00001238039878, 0., -0.00001032046688, 0., -0.000008693366246, 0.,
-0.000007391121100, 0., -0.000006336534309]

```
> U:=a0+b0*t+sum(A[n]*cos(n*Pi*x/L)*cos(n*Pi*c*t/L),n=1..NumSol)+sum
(B[n]*cos(n*Pi*x/L)*sin(n*Pi*c*t/L),n=1..NumSol);
```

U := -0.000006336534309 cos(40 π x) sin(40 π t)
- 0.00005078756070 cos(20 π x) sin(20 π t)
- 0.00003814085586 cos(22 π x) sin(22 π t)
- 0.00002936845902 cos(24 π x) sin(24 π t)
- 0.00002309314726 cos(26 π x) sin(26 π t)
- 0.00001848589374 cos(28 π x) sin(28 π t)
- 0.00001502724265 cos(30 π x) sin(30 π t)
- 0.00001238039878 cos(32 π x) sin(32 π t)

$$\begin{aligned}
& -0.00001032046688 \cos(34 \pi x) \sin(34 \pi t) \\
& -0.000008693366246 \cos(36 \pi x) \sin(36 \pi t) \\
& -0.000007391121100 \cos(38 \pi x) \sin(38 \pi t) \\
& -0.008737121565 \cos(40 \pi x) \cos(40 \pi t) + \frac{2t}{\pi} \\
& -0.06754745574 \cos(2 \pi x) \sin(2 \pi t) \\
& -0.006754745574 \cos(4 \pi x) \sin(4 \pi t) \\
& -0.001929927307 \cos(6 \pi x) \sin(6 \pi t) \\
& -0.0008041363777 \cos(8 \pi x) \sin(8 \pi t) \\
& -0.0004093785196 \cos(10 \pi x) \sin(10 \pi t) \\
& -0.0002361799152 \cos(12 \pi x) \sin(12 \pi t) \\
& -0.0001484559466 \cos(14 \pi x) \sin(14 \pi t) \\
& -0.00009933449373 \cos(16 \pi x) \sin(16 \pi t) \\
& -0.00006970841665 \cos(18 \pi x) \sin(18 \pi t) \\
& -0.01530117939 \cos(30 \pi x) \cos(30 \pi t) \\
& -0.01350454513 \cos(32 \pi x) \cos(32 \pi t) \\
& -0.01200382940 \cos(34 \pi x) \cos(34 \pi t) \\
& -0.01073803191 \cos(36 \pi x) \cos(36 \pi t) \\
& -0.009660952031 \cos(38 \pi x) \cos(38 \pi t) \\
& -0.03294931192 \cos(20 \pi x) \cos(20 \pi t) \\
& -0.02761123088 \cos(22 \pi x) \cos(22 \pi t) \\
& -0.02344486949 \cos(24 \pi x) \cos(24 \pi t) \\
& -0.02013867138 \cos(26 \pi x) \cos(26 \pi t) \\
& -0.01747546553 \cos(28 \pi x) \cos(28 \pi t) \\
& -0.07865472263 \cos(12 \pi x) \cos(12 \pi t) \\
& -0.06167055060 \cos(14 \pi x) \cos(14 \pi t) \\
& -0.04916751502 \cos(16 \pi x) \cos(16 \pi t) \\
& -0.03991291451 \cos(18 \pi x) \cos(18 \pi t) \\
& -0.09519575347 \cos(6 \pi x) \cos(6 \pi t) \\
& -0.1230769119 \cos(8 \pi x) \cos(8 \pi t) \\
& -0.1008636216 \cos(10 \pi x) \cos(10 \pi t) \\
& -0.01750258988 \cos(2 \pi x) \cos(2 \pi t) \\
& +0.4280627300 \cos(4 \pi x) \cos(4 \pi t) + \frac{225}{448}
\end{aligned}$$

```
> #The animated solution!  
animate(plot,[U,x=0..L],t=0..2,frames=100);
```

$t = 0.$

