Summary of Chapter 6 (Transforms)

In general, we should be able to show that certain given table entries are true, transform certain ODEs and PDEs, and solve for the transformed solution. We won't do much in computing the inverse transform (that way, you won't need a lot of tables).

Be sure you understand why we use transforms and the general process.

Laplace transform

- Typically used to transform time (t to s).
- Be sure to transform the boundary conditions as well.
- Be able to transform a PDE (heat or wave) using the Laplace transform.
- Be able to compute the transform of a given function (exponential, t, $\sin(ax)$, Heaviside and Dirac).
- Prove the formulas for the Laplace transform of $f'(x)$ and $f''(x)$.

New functions

The functions from 6.1 are important in several fields, so it's good to know their definitions. This is the error function and the complementary error function:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-z^2}\, dz \qquad \operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

It is also important to know that $\int_{-\infty}^{\infty} \mathrm{e}^{-z^2}\, dz = \sqrt{\pi}$, and be able to sketch a Gaussian function and the error function. Understand why the error function has that constant in front of it (think of the minimum and maximum of the error function for $x > 0$).

Sine and cosine transform

- Typically used to transform x (to α) when $x > 0$ (or, the half-line).
- I won't ask anything about the derivation from the discrete series to the continuous transform.
- Be able to compute the transforms of some functions using the definitions of the sine and cosine transform, in particular, the sine/cosine transform of $f'(x)$ and $f''(x)$ in terms of the transform of $f(x)$.
- When computing the transform, sometimes it makes sense to compute both transforms at once (See exercise 6.2.1, also included below).

- Be able to transform an ODE and solve for the transformed solution. We typically will not invert the transform on the exam.
- Why does the sine/cosine transform only produce bounded solutions?
- Be able to transform a PDE and solve for the transformed solution (we typically won't be inverting the transform).

Fourier transform

- Typically used to transform x (to α) when $-\infty < x < \infty$.
- From the definition, be able to compute the transform of things like $f(x)$ from Example 1, or the exponential function from Example 2. I won't ask you to compute the transform of the Gaussian (Ex 3).
- In the section on **Transforms and Derivatives**, be able to show the derivative formulas (first and second) as we did for Laplace and sine/cosine transforms.
- For the convolution, we'll have the table, but remember that there's a constant floating around that wasn't around for Laplace.
- Be able to show the shift property:

$$\mathcal{F}(f(x-c)) = \mathrm{e}^{-ic\alpha}F(\alpha)$$

- Be able to show that the Fourier transform is a linear operator.
- Typically, for PDEs, I'll ask you to compute the transform of the solution $U(\alpha, t)$ (that is, I typically won't ask you to invert the transform).