Review Questions, Exam 2 (Fourier series)

1. True or False (and give a short reason):

   (a) If $f(x)$ is PWS on $[0, L]$, we can find a series representation for f using either sine series or a cosine series.

   (b) If $f(x)$ is PWS on $[-L, L]$, we can find a series representation for f using either sine series or cosine series.

   (c) If f is PWS on $[-L, L]$, then the sine series for $f(x)$ will converge to the odd extension of f.

   (d) The Gibbs phenomenon occurs only when we use a finite number of terms in the Fourier series to represent a function with a jump discontinuity.

   (e) The functions $\sin(nx)$ for $n = 1, 2, 3, \ldots$ are orthogonal to the functions $\cos(mx)$ for $m = 0, 1, 2, \ldots$ on $[0, \pi]$.

2. What does the Fundamental Theorem of Fourier series say? (be specific and complete!).

3. Is f periodic (if so, give the period)?

   (a) $f(x) = \cos(x/4) + \sin(x)$

   (b) $f(x) = \cos(3x) + \cos(4x)$

   (c) $f(x) = x \sin(x)$

4. Is f piecewise continuous (PWC)? Is f piecewise smooth (PWS)?

   (a) $f(x) = \begin{cases} x^2 & \text{if } -\pi < x < 0 \\ x^2 + 1 & \text{if } 0 \le x < \pi \end{cases}$

   (b) $f(x) = \begin{cases} -\ln(x-1) & \text{if } 0 < x < 1 \\ 1 & \text{if } 1 \le x < 2 \end{cases}$

   (c) $f(x) = \sqrt[3]{x}$ on $[-1, 1]$

   (d) $f(x) = |x|$ on $[-1, 1]$.

5. Prove (using the definition), that the product of an odd and even function is odd.

6. Show that x^n and x^m are orthogonal on $[-L, L]$ (using the usual inner product, and assuming n, m are positive integers) if n, m are not both even or both odd.

7. True or False? (If false, give an example) Assume f is PWS on $[-L, L]$.

   (a) If f is continuous, so is the Fourier series of f.

   (b) If f is discontinuous, so is the Fourier series of f.

8. Draw the Fourier sine series for the function (showing at least three periods):

$$f(x) = \left\{ \begin{array}{cl} 3 & \text{if } x = 0 \text{ or } x = 1 \\ x + 1 & \text{otherwise, } 0 < x \leq 2 \end{array} \right]$$

9. Draw the Fourier cosine series of the function in the previous problem (showing at least three periods).

10. Let $f(x) = 3x + 5$. Compute the even and odd parts of f.

11. Let

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 2 & \text{for } 1 < x < 2 \end{cases}$$

   (a) Write the even extension of f as a piecewise defined function.

   (b) Write the odd extension of f as a piecewise defined function.

   (c) Draw a sketch of the periodic extension of f.

   (d) Find the Fourier sine series (FSS) for f, and draw the FSS on the interval $[-4, 4]$.

   (e) Find the Fourier cosine series (FCS) for f, and draw the FCS on the interval $[-4, 4]$.

12. Let $f(x)$ be given as below.

$$f(x) = \begin{cases} x & \text{if } -1 < x < 0 \\ 1 + x & \text{if } 0 < x < 1 \end{cases}$$

   (a) Find the Fourier series for f (on $[-1, 1]$), and draw a sketch of it on $[-3, 3]$.

   (b) Find the Fourier sine series for f on $[0, 1]$ and draw a sketch of it on $[-3, 3]$.

   (c) Find the Fourier cosine series for f on $[0, 1]$ and draw a sketch of it on $[-3, 3]$.

13. In the formula for the coefficients of the Fourier series, we have either $1/L$ or $2/L$ (depending on the interval). Where did these come from? Verify these constants directly.