Topics for Exam 2

Exam 2 will cover material from Chapter 3 (except 3.5). You won't be able to use notes on this exam, however, the trigonometric identities (attached as the last page) will be provided.

Overview:

Chapter 3 covers Fourier series. We begin by looking at the properties of sine and cosine, and see that these functions can be used as "building blocks" for functions.

- Definitions:

  "Periodic with period T", fundamental period, even and odd functions,

  The inner product (of $f(x), g(x)$ on $[a, b]$), orthogonality (of $f(x), g(x)$),

  Notation: $f(x_0+)$ and $f(x_0-)$. Piecewise continuity for f (PWC), and piecewise smooth (PWS).

  Define the even extension of f, the odd extension of f, the even part of f, the odd part of f. Also the periodic extension of f.

  Define the Fourier series for $f(x)$ on $[-L, L]$ (along with how we compute the coefficients).

  Define the Fourier cosine series for $f(x)$ on $[0, L]$ (includes the formula for the coefficients). Similarly define the Fourier sine series.

- Theorems:

  The fundamental theorem of Fourier series: If $f(x)$ is PWS on $[-L, L]$, then the (full) Fourier series converges on $[-L, L]$.

  Also be able to fill in what the Fourier series converges to.

  We had a series of theorems telling us when the Fourier series is continuous:

  - For the full series.
  - For the cosine series.
  - For the sine series.

- Computations:

  Be able to compute the full Fourier series for a given function, and also the Fourier sine series and cosine series. Similarly, be able to compute the inner product between two functions, and to show that, for example, $\sin(nx)$ is orthogonal to $\cos(mx)$ on $[-\pi, pi]$.

- Other things:

  - Be able to discuss what the Gibbs phenomoenon is, and why it is discussed here (for convergence).

Trigonometric Identities for Integrals

A few identities that are handy to recall:

- $\sin^2(\theta) + \cos^2(\theta) = 1$ (Pythagorean identity)
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

Then from the last of these (and the Pythagorean identity), we get

- $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$

We'll need some others to help us integrate products:

- $\cos(mx)\cos(nx) = \frac{1}{2}(\cos((m-n)x) + \cos((m+n)x)$
- $\sin(mx)\sin(nx) = \frac{1}{2}(\cos((m-n)x) - \cos((m+n)x)$
- $\cos(mx)\sin(nx) = \frac{1}{2}(\sin((m+n)x) - \sin((m-n)x)$