Exercises to Replace Section 2.2

1. What were the three main points in putting together the heat equation?

2. Suppose we have a rod placed $0 \leq x \leq L$, and the initial temperature profile is $f(x)$. Further, the left end of the rod is insulated and the right end is held at a constant $100°$. Write the IBVP for the temperature at time t.

3. Consider a thin plate in the shape of a quarter disk of radius r whose faces (top and bottom) are insulated (so that heat can only travel in the plane). Suppose that

   - Initially the temperature profile is given by the function $f(x, y)$.
   - One edge (along the $x-$axis) is kept at constant $0°$
   - One edge (along the $y-$axis) is kept insulated.
   - The circular edge is kept at a constant $100°$.

   (Note that the quarter disk is only in the first quadrant). Write the IBVP for the two dimensional heat equation.

4. Verify that the function

$$u(x, t) = e^{-\pi^2 t/50} \sin\left(\frac{\pi x}{10}\right) - 5e^{-4\pi^2 t/50} \sin\left(\frac{2\pi x}{10}\right)$$

   is a solution to the IBVP:

$$\begin{aligned}
u_t &= 2u_{xx} \qquad 0 < x < 10, t > 0 \\
u(0, t) &= u(10, t) = 0, \quad t > 0 \\
u(x, 0) &= \sin(\pi x/10) - 5\sin(2\pi x/10) \quad 0 < x < 10
\end{aligned}$$