Review Questions, Exam 3 (Math 367, Spr 23)

The exam will be over 4.1-4.4, and 5.1-5.3. We didn't get into the "domain of dependence" in 5.3, so only D'Alembert's solution will be on the exam from that section. As a reminder, the eigenfunctions summary page will also be provided.

1. Solve, using a suitable change of coordinates: $u_t + xu_x = 1$ with $u(x, 0) = f(x)$.

2. Solve the wave equation using D'Alembert's equation:

   (a)
   $$\begin{aligned} &u_{tt} = u_{xx}, \quad -\infty < x < \infty, t > 0 \\ &u(x, 0) = x \\ &u_t(x, 0) = 1 \end{aligned}$$

   (b)
   $$\begin{aligned} &u_{tt} = 4u_{xx}, \quad -\infty < x < \infty, t > 0 \\ &u(x, 0) = 0 \\ &u_t(x, 0) = \cos(x) \end{aligned}$$

3. We'll solve the same wave equations as the previous problem, but now the interval is $x \in [0, L]$ and use the boundary conditions: $u(0, t) = u(L, t) = 0$. We'll solve this "formally", meaning that you should write down any integrals you would need to evaluate, but you may leave them unevaluated.

4. Consider
   $$\begin{aligned} &u_t = u_{xx} \qquad 0 < x < 1 \\ &u(x, 0) = f(x) \\ &u_x(0, t) = t \\ &u_x(1, t) = t^2 \end{aligned}$$
   Convert the PDE into one with homogeneous boundary conditions, and write the new PDE that one would need to solve (with the new BCs).

5. Solve the nonhomogeneous PDE below.
   $$\begin{aligned} &u_{tt} = u_{xx} + \sin(x) \qquad 0 < x < \pi \\ &u(x, 0) = \sin(3x) \\ &u_t(x, 0) = \sin(5x) \\ &u(0, t) = 0 \\ &u(\pi, t) = 0 \end{aligned}$$

6. In D'Alembert's solution to the wave equation, we started with the equation below and made the following change of coordinates:
   $$u_{tt} = c^2 u_{xx} \qquad \begin{aligned} \xi &= x + ct \\ \eta &= x - ct \end{aligned}$$
   Write down the new PDE we get in $u(\xi, \eta)$. Show your work!

7. Given $x^2u_x + yu_y + xyu = 1$, use a change of coordinates so that the new equation involves only one derivative, and write the new equation (using variables ξ, η). Do NOT solve the PDE.

8. Solve the heat equation $u_t = 4u_{xx}$ for a rod of length π with both ends insulated if $u(x, 0) = f(x)$. You may formally solve the PDE, meaning any integrals should be written down, but you can leave them unevaluated.

9. Give a formal solutionto the wave equation below (meaning write down any integrals you're computing, but you may leave them unevaluated).

$$\begin{aligned}
&u_{tt} = 9u_{xx} \\
&u(x, 0) = f(x) \\
&u_t(x, 0) = g(x) \\
&u(0, t) = 0 \\
&u_x(L, t) = 0
\end{aligned}$$