Summary: Autonomous D.E.s (2.6)

1. Definition: An ODE is said to be **autonomous** if it can be written as $y' = f(y)$.

2. Definition: An **equilibrium solution** is a *constant* solution, $y(t) = c$, where $f(c) = 0$. That is, you solve for equilibrium solutions by solving $f(y) = 0$ for y.

3. Definition: Let $y(t) = c$ be an equilibrium solution. Then it is said to be **asymptotically stable** if all nearby solutions tend towards the equilibrium as $t \to \infty$. Algebraically, this means $\frac{df}{dy}(c) < 0$. The equilibrium $y(t) = c$ is said to be **asymptotically unstable** if all nearby solutions tend away from $y = c$ as $t \to \infty$. Algebraically, $\frac{df}{dy}(c) < 0$. NOTE: If $\frac{df}{dy}(c) = 0$, these algebraic tests don't work. Look at the phase diagram to get the stability.

4. Remark: Since $y' = f(y)$, we can plot $f(y)$ versus y. Such a plot is called a **phase diagram**. From the phase diagram, it is possible to construct a sketch of $y(t)$ on the direction field. From the phase diagram, we can determine where $y(t)$ is increasing, decreasing and concave up or down.

5. Remark: Since
$$y''(t) = \frac{df}{dy}\frac{dy}{dt}$$
we can determine concavity of $y(t)$ by considering both the local slopes on the phase diagram and the sign of $y'(t)$.

6. Remark: If $y' = f(y)$, the slopes in the direction field are independent of time. Therefore, on the direction field, the slopes along any horizontal line are identical. Some implications:
   - One can determine if a direction field is consistent with an autonomous differential equation.
   - One can quickly sketch the direction field for an autonomous differential equation.
   - Solutions to autonomous first order differential equations **cannot** be periodic.

7. Note that all autonomous ODEs are seperable, so if we can find an antiderivative for $\frac{1}{f(y)}$, we can solve all autonomous first order differential equations.