Summary of Chapter 2

1. Integration Techniques to remember:
   (a) Integration by parts (with a table)
   (b) Partial Fractions
   (c) Rationalization/simplification.

2. Analytic Solution Techniques:
   (a) Integrating Factor: $y' + p(t)y = g(t)$
   (b) Seperable: $y' = f(t)g(y)$
   Note that $y' = f(t)$ and $y' = f(y)$ are in this class.
   (c) Exact Equations: $f_x + f_y \frac{dy}{dx} = 0$
   Check by verifying that $f_{xy} = f_{yx}$
   (d) Homogeneous first order: $y' = f\left(\frac{y}{x}\right)$ Use the substitution: $v = \frac{y}{x}$, or $y = vx$. Note that $y' = v'x + v$.
   (e) Autonomous DEs: $y' = f(y)$ Be able to solve for and classify equilibrium solutions (in addition to obtaining a general solution). Be able to discuss how the equilibria change with respect to a given parameter (bifurcation).

3. Existence and Uniqueness Theory
   (a) For linear ODEs: Let $y' + p(t)y = g(t)$. If p, g are continuous for $\alpha \le t \le \beta$, then a solution exists and is unique for $t_0 \in (\alpha, \beta)$. Furthermore, the unique solution exists in that entire time interval.
   (b) For nonlinear ODEs: Let $y' = f(t, y), \quad y(t_0) = y_0$. If f is continuous at (t_0, y_0), then a solution exists. If $\frac{\partial f}{\partial y}$ is continuous at (t_0, y_0), then that solution is also unique. The time interval is only guaranteed to be $t_0 - h \le t \le t_0 + h$, so to find the entire interval, we must actually solve the ODE.

4. Graphical Analysis.
   (a) Autonomous DEs: Be able to sketch the phase diagram, and from that, determine where the solution is increasing, decreasing, concave up and concave down.
   (b) Be able to use Maple to draw direction fields and solution curves using the `dfieldplot` and `phaseportrait` commands.
   (c) Determine the type of ODE from the Direction Field: (1) If the slopes along any vertical line are the same, the ODE has the form: $y' = f(t)$. (2) If the slopes along any horizontal line are the same, the ODE has the form: $y' = f(y)$ (so it is autonomous). (3) If the slopes along any line through the origin are the same, then the ODE has the form: $y' = f\left(\frac{y}{x}\right)$, and is homogeneous.

5. Modeling.
   (a) Know how to construct a model for the tank mixing problems. Be able to obtain and analyze solutions.
   (b) Given the model for Newton's Law of Cooling, be able to obtain and analyze solutions.
   (c) Given the model for population growth (with/without thresholds), be able to obtain and analyze solutions.
   (d) Be able to solve "half-life" problems assuming exponential growth/decay.