

Final Exam
Math 467
Spring 2009

As in the homework, you should write up the solutions to the following, and attach or send me any Matlab code you might have used to get the answer. Part of your grade will be based on neatness and completeness of your answers- It should be clear that you have spent some time thinking about each problem. Each question is worth 20 points. You may use your textbook and our class notes. The use of Maple should be limited to checking answers symbolically. You should not use the internet (except our class site and the links therein).

Due date: Tuesday May 19th, 10PM

1. (a) Use Matlab's `ode45` command to solve Exercise 7.1.4, p. 365:

$$y'' = \sin(y') \quad y(0) = 1 \quad y(1) = -1$$

(Use the shooting method and the Secant method).

- (b) Use our own Runge-Kutta order 4 code to solve the higher order Boundary Value Problem:

$$y'' - \frac{1}{2}(y^2 - 1)y' + y = 0$$

for $y(0) = 0$ and $y(2) = 1$, with a step size that is small enough so that error is less than 10^{-4} , and halving the step size gets the error less than (at least) 10^{-5} .

2. (a) Use Matlab to numerically estimate the flux integral of the vector field $\vec{F}$ through the surface of the torus, defined in parametric form below:

$$\vec{F} = \langle xz, -2y, 3x \rangle$$

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$$

where

$$\begin{aligned} x &= (2 + \cos(v)) \cos(u) \\ y &= (2 + \cos(v)) \sin(u) \\ z &= \sin(v) \end{aligned} \quad 0 \leq u \leq 2\pi, \quad 0 \leq v \leq 2\pi$$

You might recall that the flux integral will be computed as:

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) dA = \int_0^{2\pi} \int_0^{2\pi} \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) du dv$$

HINT: When using `dblquad` to integrate a function, $F(x, y)$, we write a script file for F that assumes x is a vector and y is a scalar. This part of the question is mostly about the Matlab.

- (b) Find the mass of the cone $z^2 = x^2 + y^2$ bounded by the plane $z = 2$ if it has density $\sigma(x, y, z) = \sqrt{x^2 + y^2}$. That is, write the Matlab code to evaluate the triple integral:

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 \sqrt{x^2+y^2} dz dy dx$$

(Write your code so that overall you have three calls to `quad`- Do not use `triplequad`)
In this case, your write up should include a discussion of how to do these types of triple integrals in general.

3. We wish to solve the following equation for x :

$$\int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.45$$

- (a) Solve it using Newton's Method so that the error, $|x_{i+1} - x_i|$ is below 10^{-8} , if possible. The function you iterate will involve numerical integration- Use Composite Simpson with 20 panels.
 - (b) In your solution, discuss the "worst case" error we can expect with using Composite Simpson (you may use Maple to help with derivative computations, but you should use Matlab beyond that).
 - (c) Is $|x_{i+1} - x_i|$ forward error or backwards error? Explain, and give the other error by comparing your answer with the area using Matlab's `quad` function (that is, use your computed x in Matlab's `quad` function to see if the area is indeed 0.45).
4. (Not a Matlab Problem) Given a $PA = LU$ decomposition, how does one (in general) solve the system $A\mathbf{x} = \mathbf{b}$ (be explicit). Show the steps if the factorization is:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}$$

5. (Not a Matlab Problem) Numerical Computations:

- (a) Convert the binary number 11.1001001001... to base 10.
- (b) Write $31/6$ in floating point format (double) using the IEEE rounding rule.
- (c) Compute the result in (double) floating point format: $(2 + (2^{-51} + 2^{-52})) - 2$
- (d) Is machine epsilon the value of the smallest number that can be stored? Explain.