A Formula Summary: Derivatives and Integrals

Differentiation:

- Forward (or backward) Difference Formula:

$$f'(x) = \frac{1}{h}\left[f(x+h) - f(x)\right] - \frac{h}{2}f''(c)$$

  This has type I error: $K_1 h + K_2 h^2 + K_3 h^3 + \ldots$

- Three point formula:

$$f'(x) = \frac{1}{2h}\left[f(x+h) - f(x-h)\right] - \frac{h^2}{6}f'''(c)$$

  which has a type II error: $K_1 h^2 + K_2 h^4 + \ldots$. And for an endpoint:

$$f'(x) = \frac{1}{2h}\left[-3f(x) + 4f(x+h) - f(x+2h)\right] + \frac{h^2}{3}f'''(c)$$

- Second Derivative, three point centered difference:

$$f''(x) = \frac{1}{h^2}\left[f(x-h) - 2f(x) + f(x+h)\right] + \frac{h^2}{12}f^{(iv)}(c)$$

  Forward Difference (see the Maple worksheet on our class website)

$$f''(x) = \frac{1}{h^2}\left[2f(x) - 5f(x+h) + 4f(x+2h) - f(x+3h)\right] + O(h^2)$$

Integration

- Trapezoidal Rule:

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[y_0 + y_1\right] - \frac{h^3}{12}f''(c)$$

  Composite Form, with $m+1$ points (m intervals), $h = (b-a)/m$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[y_0 + y_m + 2\sum_{n=1}^{m-1} y_n\right] - \frac{(b-a)h^2}{12}f''(c)$$

- Simpson's Rule:

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[y_0 + 4y_1 + y_2\right] - \frac{h^5}{90}f^{(iv)}(c)$$

  Composite Form using $2m+1$ points, $a = x_0$, $b = x_{2m}$, evenly spaced:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[y_0 + y_{2m} + 4\sum_{i=1}^{m} y_{2i-1} + 2\sum_{i=1}^{m-1} y_{2i}\right] - \frac{h^4}{180}(b-a)f^{(iv)}(c)$$