Homework Exercises: Set 2 DUE FRIDAY, Feb 13th

1. Suppose we wish to use Newton's Method to solve for a root of function $f(x)$, and say that root is $x = r$.

   The iteration is then:

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \doteq N(x_n)$$

   From this formula, it is not clear that if r is a double root for f, then do we even have convergence?

   HOMEWORK: Show that if f has a root of multiplicity 2 at $x = r$, then Newton's Method will still converge locally to the root, but only linearly. HINT: Use the new form of f, then show that $N'(r) = 1/2$.

2. Consider the function

   $$f(x) = \frac{1}{5}(x+3)(x-2)^2$$

   Apply Newton's Method to each root.

   - For the root $x = -3$, confirm the quadratic convergence by finding (book notation):
     $$\lim_{n\to\infty} \frac{e_{n+1}}{e_n}$$
     and compare it to the theoretical limit.
   - For the root $x = 2$, show that the convergence is indeed only linear. Use the speed-up shown in class and show that you now get quadratic convergence. ("Show" in this case means to compute e_{n+1}/e_n^2 or e_{n+1}/e_n, and see that the limit is approaching what it is supposed to approach).

3. (Ex 7, p 16) Let $fl(x)$ be defined as the floating point representation of x. Show that
   $$(fl(7/3) - fl(4/3)) - 1$$
   is machine epsilon, not zero. Verify your answer on Matlab.

4. Consider the following Matlab code:

```
y=1;
n=15;

z=input('Enter z: ');

for i=1:n
  y = 2*y/3 + z/( 3*y^2 );
end
```

   Although we can run the code, we want to use our theory to prove whether or not this algorithm will converge, what it will converge to, and for what z. (HINT: Think FPI, where z is a constant)

5. Look up the `input` command, and run the code for $z = 123$. Before you run it, modify the code to keep track of e_{n+1}/e_n , and verify that it goes to $|g'(r)|$.