Notes on Exam 1

Exam 1 will cover the material from Chapters 0, 1, and sections 2.1-2.2.

There will be two parts: One part of the exam will be a take home exam. It will be posted on our class website sometime on Friday. You may use the text, class notes, and Matlab, but you should refrain from using the internet or classmates- Your work should be your own. Solutions will be due next Wednesday, March 4th at the beginning of class.

The in-class portion of the exam will be 50 minutes. There will be some "proof", some computation. You should bring a calculator with you (anything that will do basic arithmetic).

Be sure you look over the homework and the attached review questions.

A Shorthand Summary

- Horner's Method (for nested multiplication): Give an example. Why is this method used?
- Binary Numbers: Convert base 10 to binary, convert base 10 to hex (or from binary to hex). Convert binary to base 10.
- Floating point representations: All of our computations will assume the "double" format- 64 bits total. Know the IEEE rounding rule.
- Good examples: Example 0.2, 0.3, 0.4. Exercises in Section 0.3: 3(a)(b), 7(a) (also done in the homework).
- Loss of significance: Avoid the subtraction of nearly equal numbers, if possible (we looked at a couple of ways of doing that).
- Review of Calculus: See homework set 1, and the current set.
- Vocab: Absolute error, relative error, "x_c is correct to within p decimal places of r", fixed point, convergence of order α, locally convergent, forward and backward error for root finding, error magnification factor (I won't ask you to memorize the sensitivity formula for roots), root of multiplicity m, flop, lower triangular, upper triangular.
- Algorithms: Bisection, FPI, Newton's Method, Modified Newton's Method, Secant Method, Method of False Position, Inverse Quadratic Interpolation (You do not need to memorize this one), LU decomposition (be able to perform LU decomposition on a matrix by hand, like we did in class)

  A note about Bisection, FPI and Newton's Method: We were able to do a lot of analysis with these algorithms- Be sure you take a look on the review sheet.
- Order of Convergence (and the limits) for the algorithms (See Table 1)

Method	**Order of convergence**	**Limit**
Bisection	Linear	$e_{i+1} \approx \frac{1}{2}\mathrm{e}_i$
Fixed Point Iteration	$\lvert g'(r)\rvert > 0$, Linear	$e_{i+1} \approx \lvert g'(r)\rvert \mathrm{e}_i$
$x_{n+1} = g(x_n)$	$\lvert g'(r)\rvert = 0$, Quadratic	$e_{i+1} \approx \dfrac{\lvert g''(r)\rvert}{2}\mathrm{e}_i^2$
Newton's Method	$f'(r) \neq 0$, Quadratic	$e_{i+1} \approx \dfrac{\lvert N''(r)\rvert}{2}\mathrm{e}_i^2$
$x_{n+1} = N(x_n)$		$e_{i+1} \approx \left\lvert \dfrac{f''(r)}{2f'(r)} \right\rvert \mathrm{e}_i^2$
$f(x) = 0$	$f'(r) = 0$, Linear	$e_{i+1} \approx \dfrac{m-1}{m}\mathrm{e}_i$
	m =multiplicity	
Secant Method	Superlinear	$e_{i+1} \approx M\mathrm{e}_i^{1.6}$
	(Multiple Root- Linear)	

Table 1: Table of Algorithms and Convergence Properties

We had a more general definition than the text: Sequence $p_n \to p$ with order α if there are non-zero positive constants λ, α, so that

$$\frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lambda$$

- Gaussian elimination- Recall how to perform row reduction, and the relationship between row operations and elementary matrices (that is, every row operation can be defined using left multiplication by an appropriate elementary matrix).