Take Home Exam 1: Math 467, Spring 2009

Instructions: You may use the text, class notes (and the class website), and Matlab (and the code you have written for the class), but you should refrain from using the internet or classmates- Your work should be your own. Solutions will be due next **Wednesday, March 4th** at the beginning of class.

1. (10 points) Calculate $\sqrt{a^2+b^2}-a$ to 4 correct digits, if

$$a = 98765432123456789 \qquad \text{and} \qquad b = 3.14159$$

   Write your answer as a Matlab script file, then "Publish to HTML". Answer the question by printing this result.

2. (30 points) The function $f(x) = \tan(\pi x) - 6$ has an exact zero (find its value). Let $x_0 = 0$ and $x_1 = 0.48$, and use 10 iterations of each of the following methods to approximate this root. Which method is most successful and why? (Write a script file to answer this question, and answer by using the "Publish to HMTL", and print). The three methods to compare:

   (a) Bisection

   (b) Method of False Position

   (c) Secant Method

3. (20 points) Let $f(x) = 1/(1+x^2)$. Suppose we use a Taylor expansion of order 4 to approximate f (based at $x = 0$).

   - Write the Taylor polynomial with remainder (Hint: The remainder will involve the 5th derivative of f).
   - Find the c guaranteed by Taylor's Theorem (on page 21), if $x = 0.8$ (and the $x_0 = 0$).

     NOTE 1: You may use Maple to help you with the derivatives.

     NOTE 2: You will need to use an equation solver to find c. Use your favorite algorithm.

   Write up your solution either in a word processor or by hand. In the word processor, copy and paste the Matlab that you use. If you write the solution by hand, attach the script file.

4. (20 points) Finish the LU decomposition program that we started in class. In particular, we constructed the matrix U, and kept the constants C in an array. How should we use the constants C to construct L? (You can check your answer with the LU decomposition we did in class). For your convenience, the code we constructed in class is online.

5. (20 points) (Non-Matlab problem) In fixed point iteration, we know that if r is the fixed point for $x_{n+1} = g(x_n)$, and if $0 < |g'(r)| < 1$, then e_{n+1}/e_n converges to $|g'(r)|$ (linear convergence). If $g'(r) = 0$, and $g''(r) \neq 0$, we have quadratic convergence and e_{n+1}/e_n^2 converges to $|g''(r)|/2$. Under what conditions will we have **cubic** convergence, and what will e_{n+1}/e_n^3 converge to? (You'll need to prove your answer, not just provide it- As a hint, you might consider how we proved quadratic convergence).