Math 467
Homework Set 3

1. Code up Algorithm 6.2. Use it to solve the following problem.

   Let A be the fifth order Hilbert matrix- that is, A is 5×5, and

   $$A(i,j) = \frac{1}{i+j-1}$$

   (In Matlab, `A=hilb(5)`)

   Solve $A\mathbf{x} = \mathbf{b}$ for a suitable $\mathbf{b}$ to find the last column of A^{-1}. Compare your answer with the exact values, found by: `B=invhilb(5)`

2. Section 6.3, 11. Look at problem 10 so we can discuss matrix partitioning in class on Thursday.

3. Write up the code given in class for the LU decomposition.

4. Write a Matlab function so you can input $L, U, \mathbf{b}$, and output $\mathbf{x}$ from $A\mathbf{x} = \mathbf{b}$, where $A = LU$.

5. Use the previous code to do the following:

   Consider a steel plate where we indicate temperatures along the exterior in the Figure. At a steady state, the temperature of Node i is equal to the average of its direct neighbors. For example,

   $$N_1 = \frac{0 + N_3 + N_2 + 5}{4}$$

   (a) Find the system of equations that will determine the steady state temperatures, $N_1, N_2, \ldots, N_8$. Note the form of the coefficient matrix.

   (b) Find the LU factorization of the coefficient matrix, again noting the forms of L, U.

   (c) Use the LU factorization to obtain the solution.

   (d) Compute A^{-1} on Matlab, and compare its form to L, U. Which is preferable to use, especially if the coefficient matrix was very large?