Math 467
Homework Set 4

1. Let A have a full set of eigenvalues, and denote a generic one by λ_A.

    (a) Write the eigenvalues of A^k in terms of λ_A?

    (b) Let $\hat{\lambda}$ be the eigenvalues of $A + \alpha I$, for some fixed value of α. Write $\hat{\lambda}$ in terms of λ_A.

    (c) What is the relationship between the eigenvalues of A and cA, for some scalar c? That is, write the eigenvalues of cA in terms of λ_A.

2. Compute the Singular Value Decomposition (SVD) of an image in Matlab (the clown). Type `help svd` for more information. Use a 4×4 array of figures to compute:

    (a) $A_1 = \sigma_1 u_1 v_1^T$

    (b) $A_5 = U(:, 1:5) * \Sigma(1:5, 1:5) * V(:, 1:5)^T$

    (c) $A_5 = U(:, 1:20) * \Sigma(1:20, 1:20) * V(:, 1:20)^T$

    (d) $A_r = U(:, 20:200) * \Sigma(20:200, 20:200) * V(:, 20:200)^T$

    Attached is an example of the Matlab code to generate an array of figures, and how to load the clown image.

3. From class, we gave conditions for the convergence of a function, $Tx + c$, to its fixed point. Suppose we are solving $Ax = b$ for x, and we create the fixed point equation by:

    $$x = x - Ax + b \quad \Rightarrow \quad x = (I - A)x + b$$

    What are the conditions for convergence in terms of the matrix A, and what is the fixed point? Does it solve the original equation?

4. Show that if A is strictly diagonally dominant, then $\|T_j\|_\infty < 1$. Hint: We showed in class the result of left multiplication by a diagonal matrix- You might need that.

5. Compare the factorizations discussed in class. Use your paper to discuss the factorization, and turn in Matlab printouts for your experiments.

    (a) The LU factorization: What is it, and how is it used?

    (b) The Schur factorization: What is it, and how is it used? To get a feel for it, try taking the Schur factorization of some arbitrary matrices (for example, a 10x10 arbitrary matrix is done by `A=randn(10,10)` Try it with some random, symmetric matrices (i.e., `AA^T` or `A^TA`). What is the difference, if any? The Matlab command is `schur`

(c) The eigenvalue/eigenvector decomposition: What form does it take for an arbitrary matrix? Use Matlab to try it with several random matrices. Try it with some random symmetric matrices. The command is `[U,D]=eig(A)`

(d) The SVD: Try it with some random matrices (non-square) The command is `[U,S,V]=svd(A)` On paper, use the SVD to solve $Ax = b$.

6. Verify the Gerschgorin Circle Theorem for an arbitrary matrix in Matlab. Try plotting the circles together with the eigenvalues. Some Matlab hints are included below.

7. Use the Gerschgorin Circles to give conditions for which we can say a symmetric matrix A is positive definite. (Hint: You can use the theorem that says it must have all positive eigenvalues).

Here is some code that will load the clown image, give the proper color table, and generate a 4x4 array of figures.

```
load clown       %The image is in array X
figure           %Opens a new figure
image(X)         %Displays clown
colormap(map)    %Puts in the right colors
figure           %Opens a new figure
subplot(2,2,1)  %Gives a 2x2 array, makes 1st one active
%To access the other 3 plots, use subplot(2,2,i)
```

Here is some code that will generate some circles and plot them (using complex numbers). To overlay plots, use the command `hold on`

```
t=linspace(0,2*pi,150);  %Use 150 points to plot
                         %They will be evenly spaced between 0 and 2Pi
z=exp(i*t);              %Forms the unit circle
a=0.5+0.8*i; radius1=0.5;
z1=radius1*z+a;          %z1 is a shifted/scaled circle
plot(real(z),imag(z),'k',real(z1),imag(z1),'k')  %Plot them both
axis equal
%Add in the centers:
Centers=[0, a];
hold on
plot(real(Centers),imag(Centers),'k*')
hold off
%Other commands you might find helpful
%To remove the diagonal elements of a matrix A
%diag(A) is a vector of the diagonal elements of A
%diag(diag(A)) is a diagonal matrix using diagonals from A
A1=diag(diag(A))-A;
%To take the absolute row sums of the result:
sum1=sum(abs(A1'))
```