

Numerical Analysis: Exam 1

Instructions: Write complete solutions to the following. If you use Matlab, attach the relevant scripts/pictures. You may also turn in a transcript of a live session- See the instructions below.

You may use your textbook and notes, but you are expected to work on your own.

The solutions are due by Friday, Oct 11, at noon.

1. Find the roots of the polynomial:

$$x^5 - 1.11078x^4 - 9.20405x^3 + 13.32421x^2 + 0.22899x - 1.97397$$

by using the eigenvalue solver “Simultaneous Iteration”. HINT: The relevant matrix is called the “companion matrix”, which is described in our textbook.

2. Compare and contrast the following matrix factorizations: QR, LU, Schur, SVD, Eigenvalue/Eigenvector (PDP^{-1}). Be sure to explain what each is, and for what matrices and methods will produce the same thing.
3. Recall that $\rho(A)$ is the *spectral radius* of a matrix A . Does $\rho(A)$ satisfy any of the properties of a matrix norm? Hint: You might see if you can construct A, B so that $\rho(A + B) > \rho(A) + \rho(B)$.
4. Download the Matlab script file `exam1matrixA.m` on my web site, and follow the instructions. This file will construct a matrix A and a vector b , and you will use Jacobi iteration to solve for x . Turn in your script file and your estimated solution x by following the instructions on `exam1matrixA.m`
5. Use Theorem 2.2, p. 56, to show that $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}, 1]$. Use fixed point iteration to find an approximation to the fixed point accurate to within 10^{-4} . Use Corollary 2.4 (p. 62) to estimate the number of iterations required to achieve this accuracy, and compare the results.
6. Give definitions of the following:
 - (a) The sequence converges to p of order α .
 - (b) The sequence is quadratically convergent.
 - (c) The sequence converges to p with rate of convergence $O(\beta_n)$

Turning in Matlab Sessions

If you want to do your work “live” rather from a script, you can turn in the Matlab session. BEFORE you begin, type `diary exam1session.txt` As you type, your commands will be transferred to a text file named `exam1session.txt`, which you can edit with any text editor (to get rid of errors or extraneous material).

If you want to turn the diary off, type `diary off`