

Matlab's Numerical Integration Commands

The relevant commands we consider are `quad` and `dblquad`. See the Matlab help files for other integration commands. By the way, “quad” refers to “adaptive quadrature”.

To integrate: $\int_a^b f(x) dx$, we use the command:

```
Q=quad(function, a, b);  
Q=quad(function, a, b, tol);
```

The second version of the `quad` function is used if there is a user-defined tolerance. The default tolerance is 10^{-6} .

The only slightly tricky thing here is in how to define and pass in the function $f(x)$. We have several options, depending on the complexity of your definition of $f(x)$. Below we give three examples:

Example:

$$\int_0^2 \frac{1}{2x^3 - 2x - 5} dx$$

which in Matlab would be:

```
f1=inline('1./(2*x.^3-2*x-5)');  
Q=quad(f1,0,2);
```

We could also define the function using “anonymous handles” (See Matlab documentation for more):

```
f2=@(x)1./(2*x.^3-2*x-5);  
Q=quad(f2,0,2);
```

We could also define the function using an M-file. Here is a sample, which we'll save as `myfun.m`:

```
function y=myfun(x)  
y=1./(2*x.^3-2*x-5);
```

In the Matlab command window, we would type:

```
Q=quad(@myfun,0,2);
```

Similarly, if you're integrating using a built-in function like:

$$\int_0^5 \sin(x) dx$$

you would type: `Q=quad(@sin, 0, 5);`

Double Integrals

Suppose we wish to integrate: $\int_1^2 \int_0^3 x^2 y dx dy$

As before, we can use either an inline function, an anonymous function handle, or an M-file:

- Using an inline function. When defining a function of more than one variable, we can list the order in which they should appear at the end:

```
f1=inline('x.^2.*y','x','y');  
Q=dblquad(f,0,3,1,2);
```

- Using an anonymous function handle:

```
f2=@(x,y)x.^2.*y;  
Q=dblquad(f2,0,3,1,2);
```

- Using an M-file. First type and save the M-file (here, we call it myfun.m again):

```
function z=myfun(x,y)  
z=x.^2.*y;
```

Then call it in Matlab using @

```
Q=dblquad(@myfun,0,3,1,2);
```

The double quadrature formula only works on rectangular regions in the plane. How would we integrate over non-square regions? It depends on the complexity of the region. Here are two options- The first is where the region is somewhat simple (a circle in the plane), and the second is more general.

1. The volume of a hemisphere: $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-(x^2+y^2)} dy dx$

In this case, the integrand is somewhat simple, and we can extend the definition of the integrand so that it is zero outside the area of interest. Mathematically speaking,

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-(x^2+y^2)} dy dx = \int_{-1}^1 \int_{-1}^1 G(x, y) dy dx$$

where

$$G(x, y) = \begin{cases} \sqrt{1-(x^2+y^2)} & \text{if } x^2+y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

In Matlab, the expression `(x.^2+y.^2<=1)` is a logical statement- It returns “1” if the expression is TRUE, “0” if FALSE. Therefore, to find the volume, we could type:

```
f3=@(x,y)sqrt(1-(x.^2+y.^2)).*(x.^2+y.^2<=1);
Q=dblquad(f3,-1,1,-1,1);
```

2. Alternatively (or more generally), given:

$$\int_a^b \int_{y=g_1(x)}^{y=g_2(x)} F(x, y) dy dx = \int_a^b \left(\int_{g_1(x)}^{g_2(x)} F(x, y) dy \right) dx = \int_a^b G(x) dx$$

That is, given a numerical value of x , we would write the function $G(x)$ as:

$$G(x) = \int_{g_1(x)}^{g_2(x)} F(x, y) dy$$

Here’s a specific example: $\int_0^2 \int_{x^2}^{2x} x^2 + y^2 dy dx$

We write the M-file for the inside integral:

```
function z=myfun(x)
n=length(x);
z=zeros(size(x));
for j=1:n
    a=x(j)^2;
    b=2*x(j);
    h=@(y)(a+y.^2);
    z(j)=quad(h,a,b);
end
```

In Matlab, type: `Q=quad(@myfun,-1,1)`; and you may get some warnings, but we do get an answer (the exact value is $216/33$).

3. Here's another example of this second type, where we build a function to evaluate the inside integral:

Evaluate the volume of the solid under the surface $z = x^3y^4 + xy^2$ and above the region bounded by the curves $y = x^3 - x$ and $y = x^2 + x$ for $x \geq 0$.

Doing our normal calculus thing, we see that the region of integration is for x ranging from 0 to 2, the parabola is the top function and the cubic is the bottom function:

$$\int_0^2 \int_{x^3-x}^{x^2+x} x^3y^4 + xy^2 dy dx \approx 961.1809$$

Now write the M-file for the integrand. Think of the expressions in x as representing parameters:

```
function z=myfun2(x)
n=length(x);
z=zeros(size(x));
for j=1:n
    a=x(j)^3-x(j);
    b=x(j)^2+x(j);
    c=x(j)^3;
    d=x(j);
    h=@(y)(c*y.^4+d*y.^2);
    z(j)=quad(h,a,b);
end
```

In the command window, type `Q=quad(@myfun2,0,2)`;