Homework Assignment 1 - Aug 30, 2006

Read the problems in Schuam's Outline: 1.2-1.7, 1.13, 1.14, 1.22

Write up your solutions to the following:

1. Convert the following base 10 numbers to binary: (a) 9.5, (b) 44/7
2. Convert the following binary numbers to decimal (fractions preferred): (a) 1101.0111 (b) $0.01\overline{10}$ (c) $0.\overline{1}$
3. Convert 9.4 to floating point binary (use IEEE rounding) Find the error in the approximation. (Note: We did this in class for 0.4).
4. In this question, we want to examine the arithmetic of floating point numbers (all our calculations here will be double precision and IEEE rounding). Suppose we store the number 9.4. Subtract 9, and store the result. Subtract 0.4 from that result. Do we get zero?
   (a) Try to predict what will happen using your results for the previous question and the representation we gave in class for 0.4.
   (b) In Matlab, type the following and record the results. Compare the final output with your prediction.

```
format long
x=9.4
y=x-9
z=y-0.4
```

5. (Calculus Review) Recall Taylor's Theorem with Remainder[1]: Let x, a be real numbers, and let f be $k+1$ times continuously differentiable on the interval between x and a. Then there exists c in the interval such that:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$
$$+\frac{f^{(k)}(a)}{k!}(x-a)^k + \frac{(x-a)^{k+1}}{(k+1)!} f^{(k+1)}(c)$$

   (a) Find the Taylor polynomial of degree 4 for $f(x) = x^{-2}$ about $a = 1$.
   (b) Use the previous result to approximate $f(0.9)$ and $f(1.1)$.
   (c) Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations in (b).
   (d) Use a calculator (or Matlab) to compare the actual error in each case with the error bound.

[1] This is the Lagrange form of the remainder- there are other formulas that we could use.