In-Class Example: Function Iteration and Error Buildup

In this example, define

$$T(x) = \begin{cases} 3x & \text{if } x < 1/2 \\ 3 - 3x & \text{if } x > 1/2 \end{cases}$$

Define the *orbit* of a point x_0 under T as the sequence of numbers

$$\{x_0, T(x_0), T(T(x_0)), T(T(T(x_0))), \ldots\}$$

Below, we implement the function T in Matlab, and we'll call it `iterate1` instead of T:

```
function y=iterate1(x)

if x<=1/2
  y=3*x;
else
  y=3-3*x;
end
```

We can compute the first 40 values of the orbit of $x_0 = 3/13$ by using a loop. In the command window, this would be:

```
x0=3/13;

for j=1:40
  y(j)=iterate1(x0);
  x0=y(j);
end
```

The orbit of 3/13 is now stored as the vector y.
Before we run the code, let's see what should happen:

$$x_0 = 3/13 \approx 0.230769$$

$$x_1 = T(x_0) = 3 \cdot \frac{3}{13} = \frac{9}{13} \approx 0.692308$$

$$x_2 = T(x_1) = 3 - 3 \cdot \frac{9}{13} = \frac{12}{13} \approx 0.923077$$

$$x_3 = T(x_2) = 3 - 3 \cdot \frac{12}{13} = \frac{3}{13}$$

Therefore, the orbit is:

$$\left\{\frac{3}{13}, \frac{9}{13}, \frac{12}{13}, \frac{3}{13}, \frac{9}{13}, \frac{12}{13}, \ldots\right\}$$

(The number 3/13 is called a *period three* point)
If we run this code, the following are the first 40 values (read from left to right):

0.692307692308	0.923076923077	0.230769230769
0.692307692308	0.923076923077	0.230769230769
0.692307692308	0.923076923077	0.230769230769
0.692307692306	0.923076923082	0.230769230753
0.692307692259	0.923076923222	0.230769230334
0.692307691001	0.923076926998	0.230769219005
0.692307657016	0.923077028953	0.230768913140
0.692306739419	0.923079781742	0.230760654775
0.692281964326	0.923154107021	0.230537678936
0.691613036808	0.925160889576	0.224517331271
0.673551993812	0.979344018564	0.061967944307
0.185903832921	0.557711498763	1.326865503711
−0.980596511134	−2.941789533402	−8.825368600205
−26.476105800615	−79.428317401844	−238.284952205533