Lab Exercises: February 18th, 2005

1. Suppose we have 5 data points,

$$(-1, 1), (0, -1), (1, -1), (2, 1), (3, 2)$$

   We have two techniques for determining a polynomial of degree 4 that interpolates these points: (1) Using linear algebra and the Vandermonde matrix, and (2) Using Lagrange Polynomials. Use Matlab and/or Maple to show that these are the same polynomial.

2. (This is not a computer question) Our textbook says something like: Given that $f(x)$ is on the interval $[a, b]$, we can always convert it to a function $g(x)$ whose domain is $[0, 1]$, and whose range values are the same as f.

   Show how to do this. That is, given $f(x)$ on $[a, b]$, what is $g(x)$? Similarly, given a $g(x)$ on $[0, 1]$, how would we find the $f(x)$ on the interval $[a, b]$?

3. The Vandermonde matrix is the matrix we get when we want to do polynomial interpolation.

   Suppose we want to interpolate a function $f(x)$ on n equally spaced points on the interval $[0, 1]$ (From the previous problem, we can always assume that the domain is $[0, 1]$).

   We want to investigate the relationship between n and the condition number (2-norm) of the Vandermonde matrix, V_n: Is there a function G so that $G(n) = \text{cond}(V_n)$? Try to find it numerically- That is, first get some data points:

$$(3, G(3)), (4, G(4)), \ldots, (20, G(20))$$

   and see if you can figure out what G is. (Hint: You might suspect that G is an exponential function)

4. Verify the results of the last row of numbers in Example 4.6, p. 67:

   The function

$$f(x) = \frac{1}{1 + \alpha^2 x^2}$$

   was approximated in the interval $[-1, 1]$ by simple interpolation at $n + 1$ points. The observed error if $n = 20$, $\alpha = 1$ was approximately 5.9×10^{-6}, but when $\alpha = 5$, was about 60.

   The error is NOT between the function and the polynomial at the interpolation points (which is always zero). Estimate the error using 1000 points in the interval $[-1, 1]$. Plot the results in a nice way.