# Short Note on Base Conversions

Is there a convenient method for converting decimal to binary? Here is one technique:

**For the integer part:** Example- Convert 49 to binary. Divide by powers of two

| | |
|---|---|
| 49/2 | 24 r 1 |
| 24/2 | 12 r 0 |
| 12/2 | 6 r 0 |
| 6/2 | 3 r 0 |
| 3/2 | 1 r 1 |
| 1/2 | 0 r 1 |

Now read the remainders from bottom to top: $(49)_{10} = (110001)_2$

**For the fractional part:** Multiply by 2, take the remaining fractional part. For example, convert 0.7 into base 2:

| | |
|---|---|
| 0.7 * 2 | 1.4 |
| 0.4 * 2 | 0.8 |
| 0.8 * 2 | 1.6 |
| 0.6 * 2 | 1.2 |
| 0.2 * 2 | 0.4 |
| 0.4 * 2 | 0.8 |

At this point, we see that this will repeat- Our number, base 2 is (read from top to bottom): 0.1011001100110011...

To go from any base to base 10, it's probably easiest to do the straight conversion. For example,

$$(110001)_2 = \left(1 \times 2^0\right) + \left(1 \times 2^4\right) + \left(1 \times 2^5\right) = 1 + 16 + 32 = 49$$

For repeating fractional parts, we could convert using a geometric series. Recall that:

$$1 + \sum_{n=1}^{\infty} r^n = \frac{1}{1-r}$$

Example: Convert the base 4 representation: 0.03030303... into base 10

$$(0.03030303...)_4 = 3\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{2n} = 3\sum_{n=1}^{\infty} \left(\frac{1}{16}\right)^{n} = 3\left(\frac{1}{1-\frac{1}{16}} - 1\right) = \frac{1}{5}$$