Flop Counts: Gaussian Elimination

For Gaussian elimination, we had the following loops:

k	j	Add/Sub Flops	Mult/Div Flops
1	$2:n = n-1$ rows	$(n-1)n$	$(n+1)(n-1)$
2	$3:n = n-2$	$(n-2)(n-1)$	$n(n-2)$
3	$4:n = n-3$	$(n-3)(n-2)$	$(n-1)(n-3)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$k+1:n = n-k-1$	$(n-k)(n-k+1)$	$(n-k+2)(n-k)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n-1$	$n:n = 1$	$1\cdot 2$	$1\cdot 3$

Recall the following summation formulas:

$$\sum_{k=1}^{m} 1 = m, \quad \sum_{k=1}^{m} k = \frac{m(m+1)}{2}, \quad \sum_{k=1}^{m} k^2 = \frac{m(m+1)(2m+1)}{6}$$

Now sum the Add/Subtract flops together:

$$\sum_{k=1}^{n-1}(n-k)(n+1-k) = \sum_{k=1}^{n-1}(n^2+n-2nk-k+k^2) =$$

$$(n^2+n)\sum_{k=1}^{n-1} 1 - (2n+1)\sum_{k=1}^{n-1} k + \sum_{k=1}^{n-1} k^2 =$$

$$(n^2+n)(n-1) - (2n+1)\frac{(n-1)n}{2} + \frac{(n-1)n(2n-1)}{6} = \frac{1}{3}n^3 - \frac{1}{3}n$$

Similarly, for the Multiply/Divide operations:

$$\sum_{k=1}^{n-1}(n-k)(n-k+2) = \sum_{k=1}^{n-1}(n^2+2n-2nk+2k+k^2) =$$

$$(n^2+2n)\sum_{k=1}^{n-1} 1 - (2n-2)\sum_{k=1}^{n-1} k + \sum_{k=1}^{n-1} k^2 =$$

$$(n^2+2n)(n-1) - (2n-2)\frac{(n-1)n}{2} + \frac{(n-1)n(2n-1)}{6} = \frac{1}{3}n^3 + \frac{5}{2}n^2 - \frac{17}{6}$$

Adding them together, the total flop count is (using big-oh notation):

$$\frac{2}{3}n^3 + O(n^2)$$