Homework 4

Math 472, Spring 2011

(Assigned on Friday, 1/28/11, Due on Tues, 2/1/11)

1. Suppose we have three values of Q, so that $Q(1) < Q(2) < Q(3)$. Show that, using the softmax update as on p. 8 of the notes (Case Study), compute the limit of $P(1), P(2), P(3)$ as $\tau \to 0$, and the limit as $\tau \to \infty$.

   SOLUTION: For the limits as $\tau \to 0$, we have:

$$P_1 = \frac{\mathrm{e}^{Q_1/\tau}}{\mathrm{e}^{Q_1/\tau} + \mathrm{e}^{Q_2/\tau} + \mathrm{e}^{Q_3/\tau}}$$

   The idea is to divide numerator and denominator by something so that we know that the exponents will be either positive or negative. If we divide numerator and denominator by $\mathrm{e}^{Q_3/\tau}$, then we get:

$$P_1 = \frac{\mathrm{e}^{(Q_1-Q_3)/\tau}}{\mathrm{e}^{(Q_1-Q_3)/\tau} + \mathrm{e}^{(Q_2-Q_3)/\tau} + 1}$$

   where each exponent is negative. As $\tau \to 0$, $(Q_1 - Q_3)/\tau$ and $(Q_2 - Q_3)/\tau$ are negative, so that the limits of each are negative infinity (so the exponential goes to zero). For the third term, $\mathrm{e}^{(Q_3-Q_3)/\tau} = 1$. Therefore,

$$\lim_{\tau\to 0} P_1 = \frac{0}{0+0+1} = 0$$

   The same thing happens with P_2, since we'll once again divide by $\mathrm{e}^{Q_3/\tau}$, and the exponents will be negative again:

$$\lim_{\tau\to 0} P_2 = \lim_{\tau\to 0} \frac{\mathrm{e}^{(Q_2-Q_3)/\tau}}{\mathrm{e}^{(Q_1-Q_3)/\tau} + \mathrm{e}^{(Q_2-Q_3)/\tau} + 1} = \frac{0}{0+0+1}$$

   And finally, for the third probability, we do the same:

$$\lim_{\tau\to 0} P_3 = \lim_{\tau\to 0} \frac{\mathrm{e}^{(Q_3-Q_3)/\tau}}{\mathrm{e}^{(Q_1-Q_3)/\tau} + \mathrm{e}^{(Q_2-Q_3)/\tau} + 1} = \frac{1}{0+0+1} = 1$$

   As for the limits as $\tau \to \infty$, we will use the same technique, but now we see that

$$\frac{Q_1 - Q_3}{\tau} \to 0, \frac{Q_2 - Q_3}{\tau} \to 0 \quad \text{and} \quad \frac{Q_3 - Q_3}{\tau} = 0$$

   Therefore, we get:

$$\lim_{\tau\to\infty} P_1 = \lim_{\tau\to\infty} \frac{\mathrm{e}^{(Q_1-Q_3)/\tau}}{\mathrm{e}^{(Q_1-Q_3)/\tau} + \mathrm{e}^{(Q_2-Q_3)/\tau} + 1} = \frac{1}{1+1+1} = \frac{1}{3}$$

$$\lim_{\tau\to\infty} P_2 = \lim_{\tau\to\infty} \frac{\mathrm{e}^{(Q_2-Q_3)/\tau}}{\mathrm{e}^{(Q_1-Q_3)/\tau} + \mathrm{e}^{(Q_2-Q_3)/\tau} + 1} = \frac{1}{1+1+1} = \frac{1}{3}$$

$$\lim_{\tau\to\infty} P_3 = \lim_{\tau\to\infty} \frac{1}{\mathrm{e}^{(Q_1-Q_3)/\tau} + \mathrm{e}^{(Q_2-Q_3)/\tau} + 1} = \frac{1}{1+1+1} = \frac{1}{3}$$

2. (Exercise 4, p. 11) Using the win-stay, lose-shift strategy, show that if the probability for machine a is never the maximum, then $P(a) \to 0$ as $t \to \infty$ (use the hint).

   To see that the hint works, we might try a few updates:

   $$P_1(a) = P_0(a) - \beta P_0(a) = (1 - \beta)P_0(a)$$

   $$P_2(a) = (1 - \beta)P_1(a) = (1 - \beta)^2 P_0(a)$$

   and so on. We see that on iteration t, we have: $P_t(a) = (1 - \beta)^t P_0(a)$. Since $\beta \in (0, 1)$, then (1-

3. (Exercise 5, p. 11)

4. Save the Matlab code for the two algorithms (4 Matlab files) and be sure you can run the scripts and interpret the output. (Nothing to turn in, we'll discuss after you have finished).

5. From your linear algebra text, summarize *the least squares problem*, and how to find the solution (we'll be working with this on Tuesday).