Homework 5 Solutions

You should try to do all of these, but only turn in the starred problems.

1. In the script file `banditW.m`, how are we modeling the payout for the machines? That is, how are we doing the actual payouts after playing?

   SOLUTION:

2. Run `banditW.m` and `banditS.m` each several times and comment on what you see being reported. Is one of the algorithms performing better than the other? (You might change the script file for `softmax` so that the two algorithms run the same number of times).

   SOLUTION:

3. (*) Let the matrix A be given below. Find a basis for the row, column and null spaces of A. You might find the `rref` command useful in Matlab, but do provide your answer by hand.

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$$

   SOLUTION: The RREF of A is given by:

$$rref(A) = \begin{bmatrix} 1 & -3 & 0 & 5 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   The column space has a basis using the first, third and fifth columns of A. The row space is spanned by the first three rows of the `rref(A)`, and the nullspace is found by solving $A\mathbf{x} = \mathbf{0}$, which in this case is two dimensional (x_2, x_4 are free variables:

$$\mathbf{x} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ 0 \\ 3/2 \\ 1 \\ 0 \end{bmatrix}$$

   These two vectors span the nullspace of A.

4. (Referring to the line of best fit) Show that what we get by solving the normal equations is the same as what we get by minimizing the error using Calculus (That is, answer questions (c) and (d) on page 2 of the handout. The error is at the bottom of page 1).

   SOLUTION:

5. (*) Exercise (j) at the top of pg. 3 will finish the work we started on Thursday- Give the solution.

   SOLUTION: The error function (divide numerator and denominator by a^2, so assume $a \neq 0$), is

$$E = \sum_{k=1}^{n} \frac{(x_k + \mu y_k)^2}{1 + \mu^2}$$

   It might be easiest to simplify the derivative of one term first. The derivative of $(x + \mu y)^2/(1 + \mu^2)$ with respect to μ simplifies to:

$$\frac{2}{(1+\mu^2)^2}(\mu^2(-xy) + \mu(y^2 - x^2) + xy)$$

   Setting the derivative to zero and distributing the sum through, we get that:

$$\mu^2 \left(-\sum_{k=1}^{n} (x_k y_k) \right) + \mu \left(\sum_{k=1}^{n} (y_k^2 - x_k^2) \right) + \left(\sum_{k=1}^{n} x_k y_k \right) = 0$$

6. (*) Find the line of best fit for the data on page 4 of the handout, and include a plot showing the data points and the line itself (publish the script file, then print it).

   SOLUTION: Sample Matlab code (let X be an array holding the data from the table)

```
[m,b]=Line1(X);
t=linspace(min(X(:,1)),max(X(:,1)));
yhat=m*t+b;
plot(X(:,1),X(:,2),'m*',t,yhat,'k-');
```

7. (*) Exercise 2 on page 6 (The eigenvalue-eigenvector problem).

   We are given that λ is an eigenvalue of A with eigenvector $\mathbf{v}$. We want to show that $1 + \beta\lambda$ is an eigenvalue of $I + \beta A$.

   Proof:

$$(I + \beta A)\mathbf{v} = \mathbf{v} + \beta A\mathbf{v} = \mathbf{v} + \beta\lambda\mathbf{v} = (1 + \beta\lambda)\mathbf{v}$$