

Common List of Topics for the Mathematics Oral Exam

1. Prove the Pythagorean Theorem.
2. Prove that $\sqrt{2}$ is irrational.
3. Prove that there are infinitely many primes.
4. Prove that the set of real numbers \mathcal{R} is uncountable.
5. Be able to do a proof by induction.
6. Discuss the formula for the sum of a geometric series.
7. Prove that the p -series diverges for $p = 1$ and converges for $p = 2$.
8. Explain how to solve a general first order differential equation.
9. Prove Euler's formula: $e^{i\theta} = \cos(\theta) + i \sin(\theta)$.
10. Know the definition of the derivative and be able to explain its geometric interpretation.
Be prepared to use the definition to find the derivatives of x^n and $\sin(x)$.
11. Derive Newton's method.
12. Know the three "value theorems" from calculus (Intermediate, Extreme and Mean) and be able to explain them geometrically.
13. Know the definition of the definite integral and be able to explain its geometrical interpretation.
14. Show that:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$