

Lagrange Method

Utility Function	U (x,y) = x * y	
Budget Constraint	I = p _x x + p _y y	with I=30, p _x =4, p _y =2
	30 = 4x + 2y	

(1) Transform Budget Constraint

$$0 = 4x + 2y - 30$$

(2) Build the Lagrangian

$$L(x,y,\lambda) = x * y - \lambda(4x + 2y - 30) \quad \lambda \text{ is known as the Lagrange Multiplier}$$

(3) Calculate partial derivatives for x, y, and λ and set them equal to zero

1. $\frac{\partial L}{\partial x} = y - 4\lambda = 0$
2. $\frac{\partial L}{\partial y} = x - 2\lambda = 0$
3. $\frac{\partial L}{\partial \lambda} = -(4x + 2y - 30) = -4x - 2y + 30 = 0$

(4) Solve for x, y, and λ

transform 3.	-4x - 2y + 30 = 0
	-4x = 2y - 30
	4x = 30 - 2y
	x = 7.5 - 0.5y
transform 1.	y = 4λ → λ = 0.25y
plug both	x - 2λ = 0
transformations into 2.	(7.5 - 0.5y) - 0.5y = 0
	7.5 = y
	y = 7.5
use that to solve for x	x = 7.5 - 0.5y
	x = 7.5 - 3.75
	x = 3.75
calculate λ	λ = 0.25y
	λ = 1.875

(5) Calculate total utility

$$U(x,y) = x * y$$

$$U = 3.75 * 7.5 = 28.125$$

Meaning of the Lagrangian Multiplier λ

λ denotes the marginal effect of an income change on utility if $L(x,y, \lambda) = xy - \lambda(p_x x + p_y y - I)$ the marginal income effect is equal to $\frac{\partial L}{\partial I} = \lambda$