Math 126: Quiz 1
September 7, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!

1. Find the area under $f(x) = 3x$ from $x = 2$ to $x = 4$ using only high-school geometry.

\[ \text{Area} = \text{Trapezoid} = \frac{1}{2} h (b_1 + b_2) = \frac{1}{2} (2)(6+12) = 18. \]
2. Let \( f(x) = x^2 + x \).

(a) Give over- and under-estimates of the area under \( f(x) \) between \( x = 1 \) and \( x = 3 \), using 4 subdivisions.  

\[
\begin{array}{c|c|c|c|c|c|c|c}
 x & 1 & 1.5 & 2 & 2.5 & 3 \\
 f(x) & 2 & 1.5 & 6 & \frac{35}{4} & 12 \\
\end{array}
\]

\[ n = 4 \rightarrow \Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \]

Overestimate (OE):
\[
\frac{1}{2} \left( 2 + \frac{15}{4} + 6 + \frac{35}{4} \right) = \frac{1}{2} \left( 8 + \frac{80}{4} \right) = \frac{82}{8} = \frac{41}{4} \text{ (10.25)}
\]

Underestimate (UE):
\[
\frac{1}{2} \left( 2 + \frac{15}{4} + \frac{35}{4} + 12 \right) = \frac{1}{2} \left( 18 + \frac{80}{4} \right) = \frac{122}{8} = \frac{61}{4} \text{ (15.25)}
\]

(b) Find this exact area by solving an appropriate definite integral.

\[
\int_{1}^{3} x^2 + x \, dx = \left. \frac{x^3}{3} + \frac{x^2}{2} \right|_{1}^{3}
\]

\[
= \left( \frac{27}{3} + \frac{9}{2} \right) - \left( \frac{1}{3} - \frac{1}{2} \right)
\]

\[
= \frac{26}{3} + 4 = \frac{38}{3} \text{ (12.667)}
\]
3. Use the Fundamental Theorem of Calculus to find

\[ \frac{d}{dx} \int_{2}^{3x^2} \frac{1}{\sqrt{t^2 + 1}} \, dt = \frac{d}{dx} \left[ F(3x^2) - F(2) \right] \]

\[ = \frac{d}{dx} F'(3x^2) - 0 \]

\[ = 6x \cdot \frac{1}{\sqrt{9x^4 + 1}} \]

4. Suppose that a particle moves with velocity given by \( v(t) = t^2 - 4t + 3 \) m/s. Find both the total distance travelled, and the net distance travelled over the first 4 seconds. (Note: You will find it helpful to plot \( v(t) \).

\[ v(t) = t^2 - 4t + 3 = (t-1)(t-3) \]

\[ \text{NET: } \int_{0}^{4} (t^2 - 4t + 3) \, dt = \left. \frac{t^3}{3} - 2t^2 + 3t \right|_{0}^{4} = \frac{64}{3} - 32 + 12 = \frac{64 - 60}{3} = \frac{4}{3} \, \text{m} \]

\[ T + 3 \, + \, \text{III} \]

\[ = \int_{1}^{4} v(t) \, dt = \left. \frac{t^3}{3} - 2t^2 + 3t \right|_{0}^{1} = \frac{4}{3} \]

\[ = \int_{0}^{1} v(t) \, dt = \left. \frac{t^3}{3} - 2t^2 + 3t \right|_{0}^{1} = \frac{4}{3} - 0 = \frac{4}{3} \]

5. Bonus: Give two examples of functions such that \( f'(x) = f(x) \). Half a point for the first, up to 1.5 points for the second.

\[ f(x) = f'(x) = e^x \quad \text{(easy!)} \]

\[ f(x) = f'(x) = 6e^x \quad \text{or} \quad xe^x \text{ or any } C e^x. \]

Note: This is a "shift" from our usual mindset of the + C.